2020 藤田医科

(1) $e^x = X$ とおく　$(X>0)$

$3X^2-7X-6=0 \iff (3X+2)(X-3)=0 \iff X=3 \quad (\because X>0)$

$e^{kx}=243 \iff 3^k = 3^5 \qquad \therefore k=5$

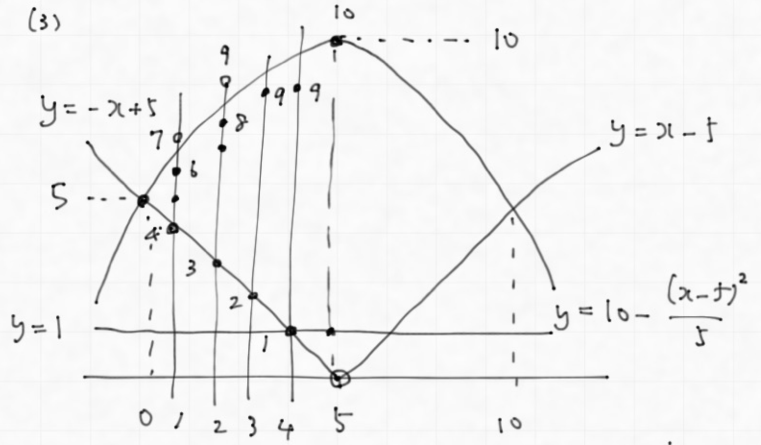
(2) $\displaystyle\int_{-2}^{2}\left(|2x-x^2|+x^2+2x+2\right)dx$

$\displaystyle = \int_{-2}^{0}-(2x-x^2)dx + \int_{0}^{2}2x-x^2\,dx + \int_{-2}^{2}x^2+2x+2\,dx$

$\displaystyle = \left[x^2-\frac{1}{3}x^3\right]_0^{-2} + \left[x^2-\frac{1}{3}x^3\right]_0^{2} + 2\left[\frac{1}{3}x^3+2x\right]_0^{2}$

$\displaystyle = +4+\cancel{\frac{8}{3}}+4-\cancel{\frac{8}{3}}+2\left(\frac{8}{3}+4\right) = 16+\frac{16}{3} = \frac{64}{3}$

(3)

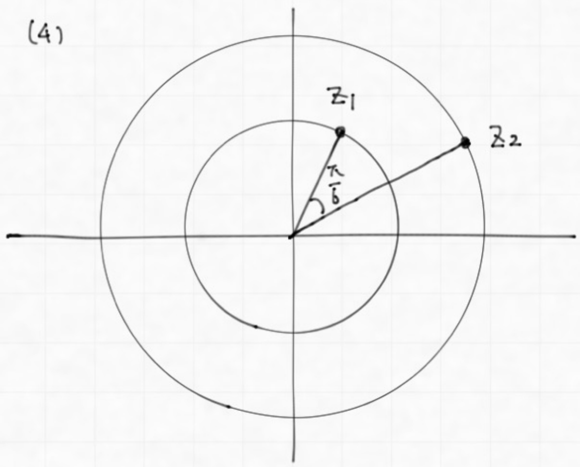


$x=0$	1コ
$x=1$	3コ
$x=2$	6コ
$x=3$	8コ
$x=4$	9コ
$x=5$	10コ

$\times 2$

64コ

(4)



$|z_1|=1$ だから

$z_1 = \cos\theta + i\sin\theta$

$z_2 = 2\left(\cos\left(\theta-\frac{\pi}{6}\right)+i\sin\left(\theta-\frac{\pi}{6}\right)\right)$

$\quad = 2\left(\frac{\sqrt{3}}{2}\cos\theta+\frac{1}{2}\sin\theta+i\frac{\sqrt{3}}{2}\sin\theta-i\frac{1}{2}\cos\theta\right)$

$z_1+kz_2 = \cos\theta+k\sqrt{3}\cos\theta+k\sin\theta$

$\qquad +i\sin\theta+k\sqrt{3}i\sin\theta-ki\cos\theta$

$|z_1+kz_2|^2 = \cancel{\cos^2\theta}+3k^2\cancel{\cos^2\theta}+\cancel{k^2\sin^2\theta}+2\sqrt{3}\cancel{k\cos^2\theta}$

$\qquad +\cancel{2k\cos\theta\sin\theta}+\cancel{2\sqrt{3}k^2\cos\theta\sin\theta}$

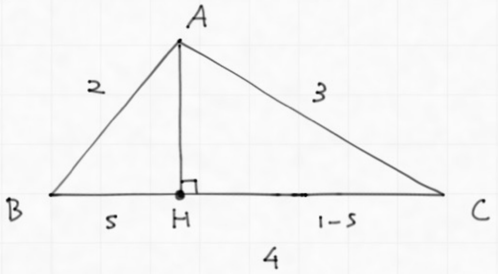
$\qquad +\cancel{\sin^2\theta}+3\cancel{k^2\sin^2\theta}+\cancel{k^2\cos^2\theta}$

$\qquad +\cancel{2\sqrt{3}k\sin^2\theta}-\cancel{2k^2\sqrt{3}\cos\theta\sin\theta}-\cancel{2k\sin\theta\cos\theta}$

$\qquad = 1+3k^2+k^2+2\sqrt{3}k$

$\displaystyle \frac{|z_1+kz_2|-|z_1|}{k} = \frac{\sqrt{4k^2+2\sqrt{3}k+1}-1}{k} = \frac{4k^{\cancel{2}}+2\sqrt{3}\cancel{k}}{\cancel{k}\left(\sqrt{4k^2+2\sqrt{3}k+1}+1\right)} \to \frac{2\sqrt{3}}{2} = \sqrt{3}$

(5)



$\overrightarrow{AB}=\vec{b}$, $\overrightarrow{AC}=\vec{c}$ とおく.

$|\vec{b}|=2$, $|\vec{c}|=3$

$|\vec{c}-\vec{b}|^2=4+9-2\cdot\vec{b}\cdot\vec{c}=16$ $\quad\vec{b}\cdot\vec{c}=-\frac{3}{2}$

$\overrightarrow{AH}=(1-s)\vec{b}+s\vec{c}$ とする

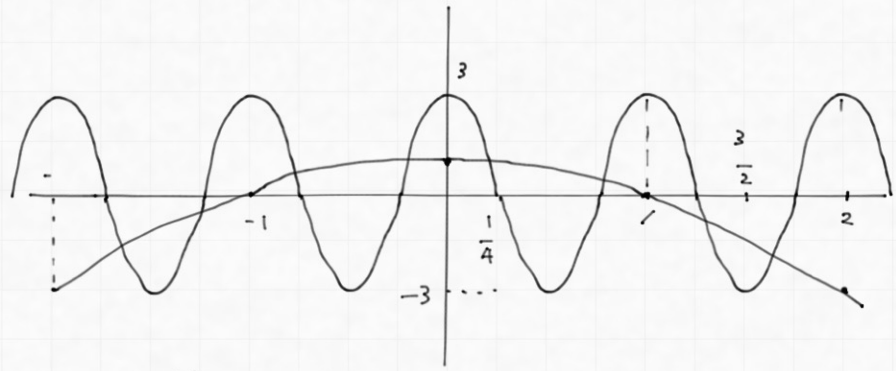
$\overrightarrow{AH}\cdot(\vec{b}-\vec{c})=(1-s)|\vec{b}|^2-(1-s)\vec{b}\cdot\vec{c}+s\vec{b}\cdot\vec{c}-s|\vec{c}|^2$

$=4(1-s)+\frac{3}{2}(1-s)-\frac{3}{2}s-9s$

$=\frac{11}{2}-16s=0 \qquad s=\frac{11}{32}$

$\overrightarrow{AH}=\boldsymbol{\frac{1}{32}(21\overrightarrow{AB}+11\overrightarrow{AC})}$

(6) $3\cos2\pi x=1-x^2$



左図より **8コ**

(7) $1, 5, 7, 11, 13, 17$ (+6, +6, +6, +6)

左の数列を $\{a_n\}$ とすると

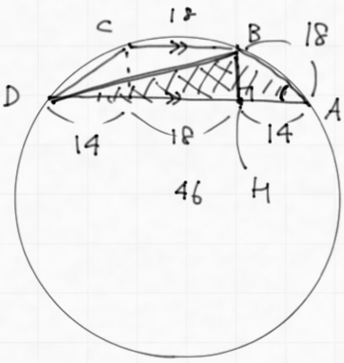
$a_{2n}=6n-1$, $a_{2n-1}=6n-5$

$a_{2020}=a_{2\cdot1010}=6\times1010-1=\mathbf{6059}$

$S=\sum_{k=1}^{1010}\{6k-1+6k-5\}=6\sum_{k=1}^{1010}(2k-1)=6\times\frac{1+2019}{2}\times1010=3\times2020\times1010$

$\frac{S}{2020}=\mathbf{3030}$

(8)



$BH=\sqrt{18^2-14^2}=\sqrt{(18+14)(18-14)}=\sqrt{128}=8\sqrt{2}$

$BD=\sqrt{\sqrt{128}^2+32^2}=\sqrt{128+32^2}=8\sqrt{2+16}=24\sqrt{2}$

$2R=\frac{24\sqrt{2}}{\sin\angle A}$ $\qquad \sin\angle A=\frac{BH}{18}=\frac{8\sqrt{2}}{18}=\frac{4}{9}\sqrt{2}$

$R=\cancel{12}^{\,3}\cancel{\sqrt{2}}\times\frac{9}{\cancel{4\sqrt{2}}}=\mathbf{27}$

(9) $(80 \times 100 + 100 \times 2) \div 100 = \mathbf{82}$

修正前の2乗平均を X とすると

$$X - 80^2 = 320$$

修正後の2乗平均は $\dfrac{X \times 100 + 100^2 \times 2}{100} = X + 200$

$$s^2 = X + 200 - 82^2$$

$$= 320 + 6400 + 200 - 6724 = 320 - 124 = \mathbf{196}$$

(10)

$$\begin{array}{r} 51.3_{(6)} \\ 24.1_{(6)} \\ \hline 115.4_{(6)} \end{array} = 6^2 + 6 + 5 + \frac{4}{6} = 47 + \frac{2}{3}$$

$9 \underline{)\,47}$ $5 \cdots 2$ 　 $\frac{2}{3} \times 9 = 6$ 　 $115.4_{(6)} = \mathbf{52.6_{(9)}}$

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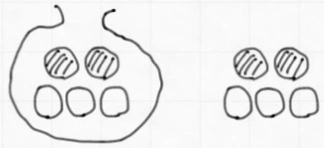
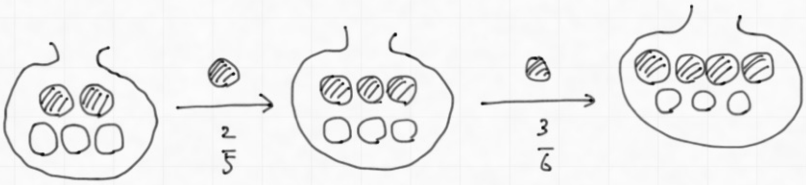
2

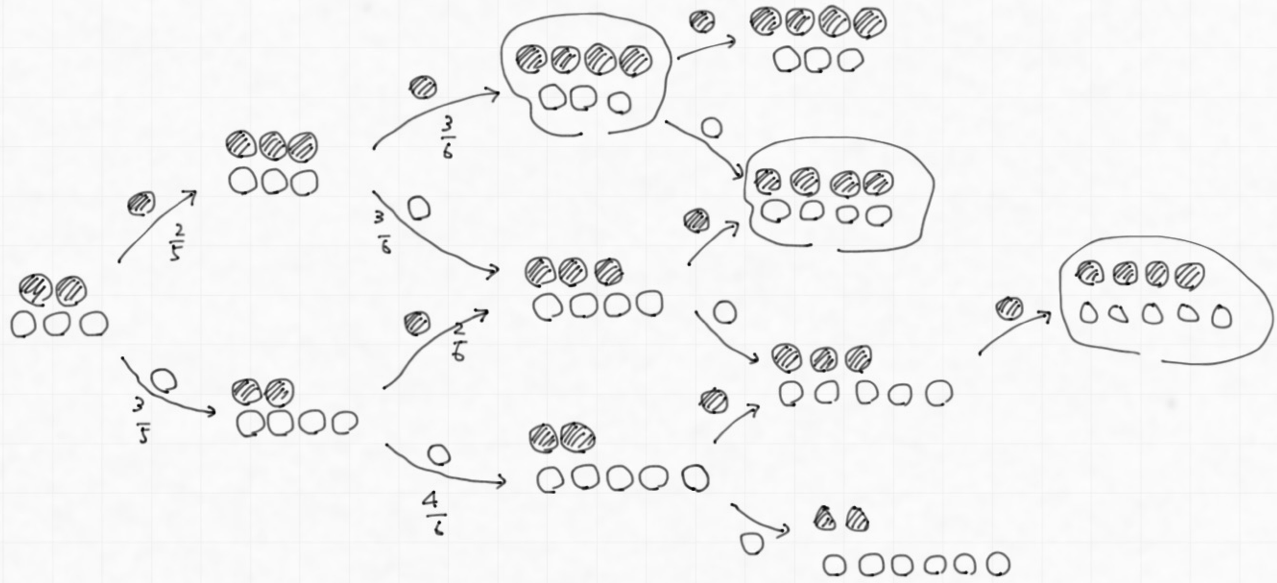


(1) 2回連続で赤玉がとり出されたことになる



$$\frac{2}{5}\times\frac{3}{6}=\frac{1}{5}$$

(2)



上図より $\frac{2}{5}\times\frac{3}{6}\times\frac{3}{7}+\left(\frac{2}{5}\times\frac{3}{6}+\frac{3}{5}\times\frac{2}{6}\right)\times\frac{4}{7}+\frac{3}{5}\times\frac{4}{6}\times\frac{5}{7}$

$$=\frac{1}{5\cdot\cancel{6}\cdot7}\left(\overset{3}{\cancel{18}}+\overset{8}{\cancel{48}}+\overset{10}{\cancel{60}}\right)=\frac{21}{5\cdot7}=\frac{3}{5}$$

(3) 上図より

$$\frac{2}{5}\times\frac{3}{6}+\left(\frac{2}{5}\times\frac{3}{6}+\frac{3}{5}\times\frac{2}{6}\right)\times\frac{3}{7}+\left\{\left(\frac{2}{5}\times\frac{3}{6}+\frac{3}{5}\times\frac{2}{6}\right)\times\frac{4}{7}+\frac{3}{5}\times\frac{4}{6}\times\frac{2}{7}\right\}\times\frac{3}{8}$$

$$=\frac{1}{5}+\frac{6}{35}+\frac{1}{5\cdot\cancel{6}\cdot7\cdot\cancel{8}_{2}}\left(\overset{2}{\cancel{48}}+\overset{1}{\cancel{24}}\right)\times3=\frac{14+12+9}{70}=\frac{1}{2}$$

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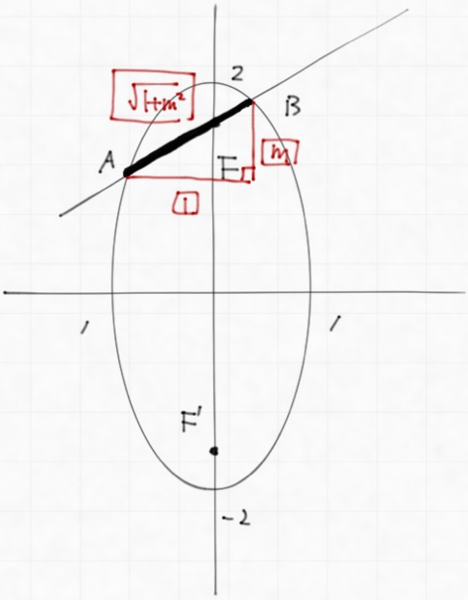


(1) $\sqrt{2^2-1} = \sqrt{3}$ だから

$$F(0, \sqrt{3}),\ F'(0, -\sqrt{3})$$

l は $\mathbf{y = mx + \sqrt{3}}$

(2) C と l の交点は

$$x^2 + \frac{1}{4}(mx+\sqrt{3})^2 = 1$$

$$\Leftrightarrow \quad (4+m^2)x^2 + 2\sqrt{3}mx - 1 = 0$$

$$x = \frac{-\sqrt{3}m \pm \sqrt{3m^2+4+m^2}}{4+m^2}$$

A, B を左のようにとり、それぞれの x 座標を α, β とすると

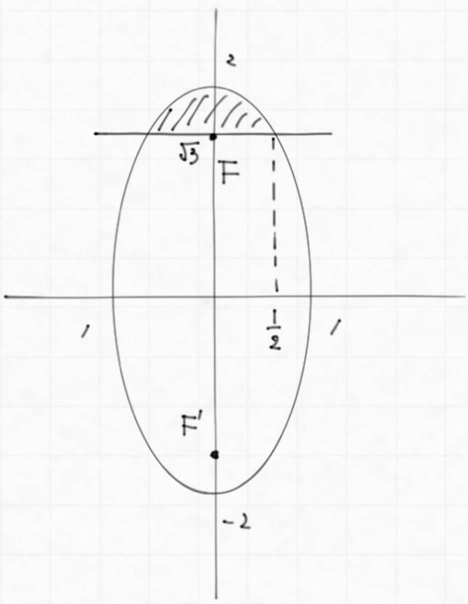
$$\beta - \alpha = \frac{-\sqrt{3}m + 2\sqrt{m^2+1}}{4+m^2} - \frac{-\sqrt{3}m - 2\sqrt{m^2+1}}{4+m^2} = \frac{4\sqrt{m^2+1}}{4+m^2}$$

$$AB = \sqrt{1+m^2}(\beta - \alpha) = \mathbf{\frac{4(m^2+1)}{4+m^2}}$$

(3) $AF' + F'B = AF' + AF + BF' + BF - AF - BF = 2\times 2 + 2\times 2 - AB = 8 - AB$

$$= 8 - \frac{4(m^2+1)}{4+m^2} = 8 - 4 + \frac{12}{4+m^2} = 4 + \frac{12}{4+m^2}$$

これが最大となるのは $m=0$ のときで、このとき $AF' + F'B = 4 + 3 = \mathbf{7}$

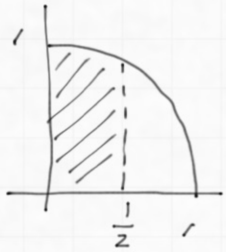


(4) 左図

$$S = \int_{-\frac{1}{2}}^{\frac{1}{2}} y - \sqrt{3}\, dx = 2\int_0^{\frac{1}{2}} 2\sqrt{1-x^2}\, dx - \sqrt{3}$$

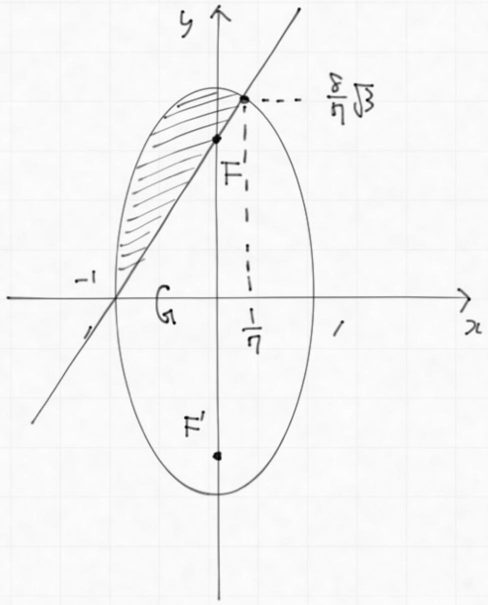
$$= 4\underbrace{\int_0^{\frac{1}{2}} \sqrt{1-x^2}\, dx}_{} - \sqrt{3}$$

～部は右図斜線部の面積に等しい



$$\pi \times 1^2 \times \frac{\frac{\pi}{6}}{2\pi} + \frac{1}{2}\times\frac{1}{2}\times\frac{\sqrt{3}}{2} = \frac{\pi}{12} + \frac{\sqrt{3}}{8}$$

$$\therefore\ S = \frac{\pi}{3} + \frac{\sqrt{3}}{2} - \sqrt{3} = \mathbf{\frac{\pi}{3} - \frac{\sqrt{3}}{2}}$$



(5)

$$x = \frac{-\sqrt{3}m \pm \sqrt{3m^2+4+m^2}}{4+m^2} \text{ より}$$

$m=\sqrt{3}$ のとき　$x = -1, \frac{1}{7}$

$x=\frac{1}{7}$ のとき　$y = \sqrt{4-4\times\left(\frac{1}{7}\right)^2} = \frac{2}{7}\sqrt{48} = \frac{8}{7}\sqrt{3}$

$$V = \int_{-1}^{\frac{1}{7}} \pi y^2 dx - \frac{1}{3}\times\pi\cdot\left(\frac{8}{7}\sqrt{3}\right)^2\times\left(1+\frac{1}{7}\right)$$

$$= \int_{-1}^{\frac{1}{7}} \pi(4-4x^2)dx - \cancel{\frac{1}{3}}\pi\cdot\frac{8^2}{7^2}\cdot\cancel{3}\cdot\frac{8}{7}$$

$$= \pi\left[4x-\frac{4}{3}x^3\right]_{-1}^{\frac{1}{7}} - \frac{8^3}{7^3}\pi$$

$$= \frac{4}{7}\pi - \frac{4}{3}\cdot\frac{1}{7^3}\pi + 4\pi - \frac{4}{3}\pi - \frac{8^3}{7^3}\pi$$

$$= \frac{\pi}{7^3\cdot 3}\left(4\cdot 7^2\cdot 3 - 4 + 4\cdot 7^3\cdot 3 - 4\cdot 7^3 - 3\cdot 8^3\right)$$

$$= \frac{\pi}{7^3\cdot 3}\left(588-4+4116-1289-1536\right) = \frac{1875}{7^3\cdot 3}\pi$$

$$= \cancel{\frac{625}{343}\pi}$$

$$\frac{256}{147}\pi$$