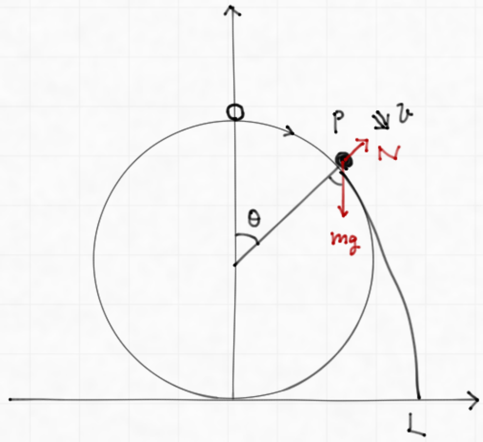
2019 大阪医大(後)



1

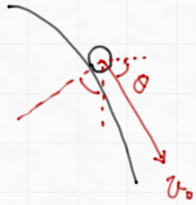
$$\begin{cases} \text{運動方程式} & m\dfrac{v^2}{a} = mg\cos\theta - N \\ \text{エネルギー保存} & 0 = \dfrac{1}{2}mv^2 - mga(1-\cos\theta) \end{cases}$$

① $N = mg\cos\theta - m\dfrac{v^2}{a}$

② $\dfrac{1}{2}mv^2 = mga(1-\cos\theta)$

③ $N = mg\cos\theta - 2mg(1-\cos\theta) = 3mg\cos\theta - 2mg$

$N=0$のとき離れることになるので　$3mg\cos\theta_0 = 2mg$　$\cos\theta_0 = \dfrac{2}{3}$



④ このとき　$\dfrac{1}{2}mv_0^2 = mga\left(1-\dfrac{2}{3}\right)$ より　$v_0 = \sqrt{\dfrac{2}{3}}\sqrt{ag}$

⑤ 水平方向成分は　$v_0\cos\theta = \dfrac{2}{3}\sqrt{\dfrac{2}{3}}\sqrt{ag} = \dfrac{2\sqrt{6}}{9}\sqrt{ag}$

⑥ 鉛直成分は　$v_0\sin\theta = \sqrt{1-\left(\dfrac{2}{3}\right)^2}\sqrt{\dfrac{2}{3}}\sqrt{ag} = \dfrac{\sqrt{30}}{9}\sqrt{ag}$

⑦ $a + a\cos\theta = \dfrac{5}{3}a$

⑧ $\dfrac{5}{3}a = \dfrac{\sqrt{30}}{9}\sqrt{ag}\,t + \dfrac{1}{2}gt^2 \iff gt^2 + \dfrac{2}{9}\sqrt{30ag}\,t - \dfrac{10}{3}a = 0$

$$t = \frac{-\frac{1}{9}\sqrt{30ag} + \sqrt{\frac{30}{81}ag + \frac{10}{3}ag}}{g} = \left(-\frac{\sqrt{30}}{9} + \frac{\sqrt{300}}{9}\right)\sqrt{\frac{a}{g}} = \frac{1}{9}\sqrt{30}\left(\sqrt{10}-1\right)\sqrt{\frac{a}{g}}$$

⑨ $OL = a\sin\theta_0 + v_0\cos\theta\cdot t = \dfrac{\sqrt{5}}{3}a + \dfrac{2\sqrt{6}}{9}\cancel{\sqrt{ag}} \times \dfrac{\sqrt{30}}{9}\left(\sqrt{10}-1\right)\sqrt{\dfrac{a}{\cancel{g}}} = \left(\dfrac{\sqrt{5}}{3} + \dfrac{60\sqrt{2}}{81} - \dfrac{12\sqrt{5}}{81}\right)a = \left(\dfrac{5\sqrt{5}+20\sqrt{2}}{27}\right)a$

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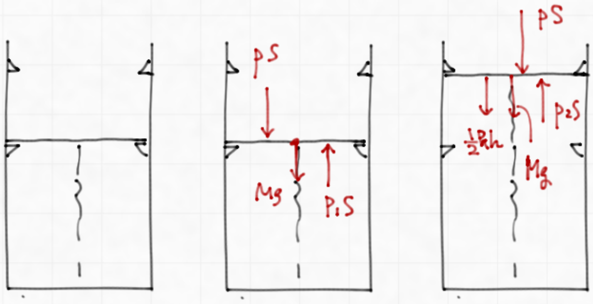
II



初期状態　　状態I

初期状態　$pSh = nRT$

定積 ↓　$Q_{01} = 0 + \frac{3}{2}nR(T_1 - T)$

状態I　$p_1Sh = nRT_1$,　$p_1S = pS + Mg$

ばね ↓　$Q_{12} = W_{12} + \frac{3}{2}nR(T_2 - T_1)$

状態II　$p_2S\cdot\frac{3}{2}h = nRT_2$,　$p_2S = pS + Mg + \frac{1}{2}kh$

① $n = \frac{pSh}{RT}$

② $p_1S = pS + Mg$ より　$p_1 = p + \frac{Mg}{S}$

③ $p_1Sh = nRT_1$ より　$T_1 = \frac{p_1Sh}{nR} = \frac{\cancel{R}T}{p\cancel{Sh}}\times\frac{\cancel{Sh}}{\cancel{R}}\left(p + \frac{Mg}{S}\right) = \left(1 + \frac{Mg}{pS}\right)T$

④⑤ $Q_{01} = \frac{3}{2}nR(T_1 - T) = n\left(\frac{3Mg}{2pS}\right)RT = \frac{3}{2}\cdot\frac{Mg}{S}\cdot Sh = \frac{3}{2}Mgh$

⑥ $p_2S\cdot\frac{3}{2}h = nRT_2$ と　$p_2S = pS + Mg + \frac{1}{2}kh$ を連立

$\frac{3}{2}\cancel{h}\left(pS + Mg + \frac{1}{2}kh\right) = \frac{pS\cancel{h}}{\cancel{R}T}\cdot\cancel{R}T_2$　　$T_2 = \frac{3}{2}\left(1 + \frac{Mg}{pS} + \frac{kh}{2pS}\right)T$

⑦ $Mg\cdot\frac{1}{2}h = \frac{1}{2}Mgh$　　⑧ $pS\cdot\frac{1}{2}h = \frac{1}{2}pSh$　　⑨ $\frac{1}{2}k\left(\frac{1}{2}h\right)^2 = \frac{1}{8}kh^2$

⑩ $\frac{3}{2}nR(T_2 - T_1) = \frac{9}{4}nR\left(1 + \frac{Mg}{pS} + \frac{kh}{2pS}\right)T - \frac{3}{2}nR\left(1 + \frac{Mg}{pS}\right)T$

$= \frac{3}{4}nR\left(1 + \frac{Mg}{pS}\right)T - \frac{9}{8}nR\frac{kh}{pS}T = \frac{3}{4}pSh\left(1 + \frac{Mg}{pS}\right) - \frac{9kh}{8pS}\cdot pSh = \frac{3}{4}\left(\left(1 + \frac{Mg}{pS}\right) - \frac{3kh}{2pS}\right)pSh$

⑪ $\frac{1}{2}Mgh + \frac{1}{2}pSh + \frac{1}{8}kh^2 + \frac{3}{4}\left(\left(1 + \frac{Mg}{pS}\right) - \frac{3kh}{2pS}\right)pSh = \frac{5}{4}\left(\left(1 + \frac{Mg}{pS}\right) - \frac{kh}{pS}\right)pSh$

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|1| ①原子質量 ②ベータ ③半減期 ④受けない ⑤中性子 ⑥CO_2

⑦減少する ⑧新しく

(1) ア, エ

(2) ア $R = R_0\left(\frac{1}{2}\right)^{\frac{T}{5730}}$ より $\frac{R}{R_0} = \left(\frac{1}{2}\right)^{\frac{T}{5730}}$

イ $\frac{R}{R_0} = \frac{1}{1024} = \left(\frac{1}{2}\right)^{10} = \left(\frac{1}{2}\right)^{\frac{T}{5730}}$ より $T = 57300 = 5.7 \times 10^4$ 年

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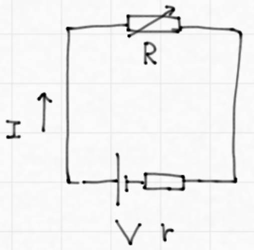
IV

(1)



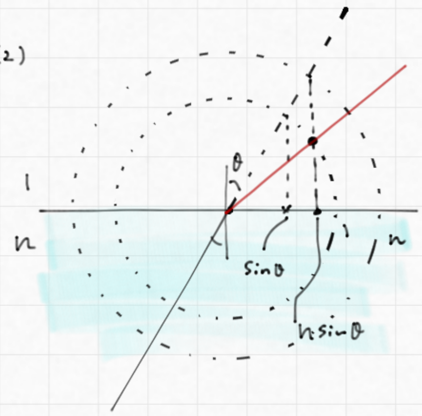
回路の式は $V = IR + Ir \iff I = \frac{V}{r+R}$

$$Q = I^2R = \frac{V^2R}{(r+R)^2} = \frac{V^2}{\frac{r^2}{R}+R+2r}$$

ここで相加相乗平均の公式より $\frac{r^2}{R}+R \geq 2\sqrt{\frac{r^2}{R}\times R} = 2r$

$\therefore Q \leq \frac{V^2}{2r+2r} = \frac{V^2}{4r}$ 　等号は $\frac{r^2}{R} = R$ すなわち $R = r$ のとき

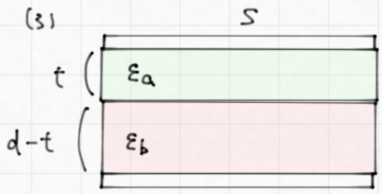
(2)



入射角をθ, 屈折角をφとして

$\frac{\sin\theta}{\sin\varphi} = \frac{1}{n}$ 　 $\sin\varphi = n\sin\theta$

(3)



上部の容量 $C_a = \varepsilon_a \frac{S}{t}$

下部 〃 $C_b = \varepsilon_b \frac{S}{d-t}$

$\frac{1}{C} = \frac{1}{C_a} + \frac{1}{C_b}$ より $C = \frac{C_aC_b}{C_a+C_b} = \frac{\varepsilon_a\varepsilon_b \frac{S^{\cancel{2}}}{t(d-t)}}{\varepsilon_a\frac{\cancel{S}}{t} + \varepsilon_b\frac{\cancel{S}}{d-t}}$

$$= \frac{\varepsilon_a\varepsilon_b S}{\varepsilon_a(d-t) + \varepsilon_b t}$$