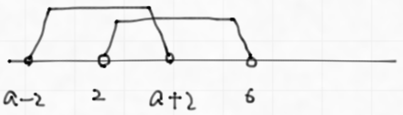
2020 同志社女子 推せん 午後

1 (1) $\sqrt{9x^2-24x+16}+\sqrt{x^2-4x+4} = \sqrt{(3x-4)^2}+\sqrt{(x-2)^2} = |3x-4|+|x-2|$

$= 3x-4+2-x = \mathbf{2x-2}$　（$\because$ $\sqrt{2}<x<2$ のとき $3x-4>0$, $x-2<0$）

(2) $6<x+4<10 \Leftrightarrow 2<x<6$

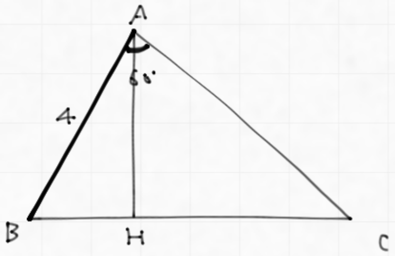


$2<a-2<6$ または $2<a+2<6$ または $a-2=2$

$4<a<8$, $0<a<4$, $a=4$

$\mathbf{0<a<8}$

(3)



$5\sqrt{3} = \frac{1}{2}\times 4\times AC\times \sin 60^\circ$　$AC = \mathbf{5}$

$BC = \sqrt{4^2+5^2-2\cdot 4\cdot 5\cdot \cos 60^\circ} = \sqrt{21}$

$\frac{1}{2}\times BC\times AH = 5\sqrt{3}$ より　$AH = \frac{10\cancel{\sqrt{3}}}{\cancel{\sqrt{21}}\ \sqrt{7}} = \mathbf{\frac{10\sqrt{7}}{7}}$

(4) 220×　${}_3C_2\times\frac{4!}{2!} = 3\times 4\cdot 3 = 36$

22××　${}_3C_1\times\frac{4!}{2!\cdot 2!} = 18$

$36+18 = \mathbf{54}$

(5)

1 0 5 0 0 7 0 0 11 0 17

1 5 5 7 7 11

$2+15+21+22+17 = 77$

$77\div 11 = 7$　$\therefore$ **7**

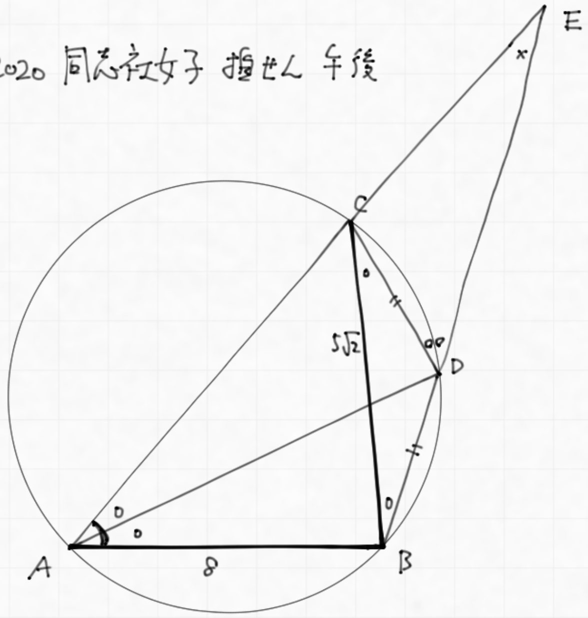
(6) $600 = 2^3\cdot 3^1\cdot 5^2 = 30\times 2^2\times 5^1$　$30\times 2^a\times 5^b$

$a = 0, 1, 2$, $b = 0, 1$

$3\times 2 = 6$　**6通り**

2020 同志社女子 推せん 午後

2



$$\sin\angle BAC = \sqrt{1-\left(\frac{9}{16}\right)^2} = \frac{\sqrt{175}}{16} = \mathbf{\frac{5\sqrt{7}}{16}}$$

$$R = \frac{BC}{2\sin\angle A} = \frac{5\sqrt{2}}{\frac{5\sqrt{7}}{8}} = \frac{8\sqrt{2}}{\sqrt{7}} = \mathbf{\frac{8\sqrt{14}}{7}}$$

$$BC^2 = AC^2 + AB^2 - 2\cdot AB\cdot AC\cdot\cos\angle BAC$$

$$50 = AC^2 + 64 - \cancel{16}\cdot\frac{9}{\cancel{16}}\cdot AC$$

$$AC^2 - 9AC + 14 = 0 \qquad AC = \mathbf{2,\ 7}$$

$BD = CD = x$ とおく $\angle BDC = 180^\circ - \angle BAC$ だから

$\triangle BCD$ で余弦定理を用いて

$$(5\sqrt{2})^2 = x^2 + x^2 - 2\cdot x\cdot x\cdot\cos(180^\circ - \angle BAC)$$

$$50 = 2x^2 - 2x^2 \times \left(-\frac{9}{16}\right) = \frac{25}{8}x^2$$

$$x^2 = 16 \qquad x = \mathbf{4}$$

$\triangle EDC \sim \triangle EAB$ より $ED : EA = EC : EB = DC : BA = 4 : 8$

$ED : EA = 1 : 2 \qquad \dfrac{AE}{DE} = \dfrac{BE}{CE} = \mathbf{2}$

$DE = y$, $CE = z$ として $AE = 2y$, $BE = 2z$

$AC = 7$ だから $\qquad AC = 7 = AE - CE = 2y - z$

$BD = 4$ 〃 $\qquad BD = 4 = 2z - y$

連立して

$$\begin{array}{rl} 2y - z &= 7 \\ 4z - 2y &= 8 \quad (+ \\ \hline 3z &= 15 \end{array}$$

$z = 5,\ y = 6 \qquad CE = \mathbf{5},\ DE = \mathbf{6}$

2020 同志社女子 推せん 午後

3 $f(x) = \frac{1}{2}x^2 - (2a+4)x + 2a^2 - 5a - 12$

(1) $= \frac{1}{2}\left(x-(2a+4)\right)^2 - \frac{1}{2}(2a+4)^2 + 2a^2 - 5a - 12$

$= \frac{1}{2}(x-2a-4)^2 - \cancel{2a^2} - 8a - 8 + \cancel{2a^2} - 5a - 12$

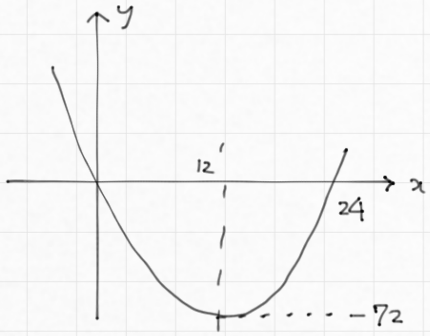
$= \frac{1}{2}(x-2a-4)^2 - 13a - 20$ $\quad$ $(2a+4, -13a-20)$ $\quad$ $2a+4=8$

$a = 2$

(2) $f(0) = 2a^2 - 5a - 12 < 0$

$(2a+3)(a-4) < 0$

$-\frac{3}{2} < a < 4$



(3) $a=4$ のとき

$f(x) = \frac{1}{2}(x-12)^2 - 72 = \frac{1}{2}x^2 - 12x$

$0 < 2t+1 \leqq 12 \iff t \leqq \frac{11}{2}$ のとき. $m = f(2t+1)$

$t \geqq \frac{11}{2}$ のとき $m = f(12) = -72$

$0 < t \leqq \frac{11}{2}$ のとき $M + m = f(-t) + f(2t+1) = \frac{1}{2}t^2 + 12t + \frac{1}{2}(2t+1)^2 - 12(2t+1)$

$= \frac{1}{2}t^2 + 12t + 2t^2 + 2t + \frac{1}{2} - 24t - 12$

$= \frac{5}{2}t^2 - 10t - \frac{23}{2} = g(t)$ $\quad$ $g'(t) = 5t - 10$

$g'(t) = 0$ のとき $t = 2$

$g(2) = 10 - 20 - \frac{23}{2} = -\frac{43}{2}$

$t \geqq \frac{11}{2}$ のとき $M + m = f(-t) + f(12) = \frac{1}{2}t^2 + 12t - 72 = \frac{1}{2}(t+12)^2$

$t = \frac{11}{2}$ で最小 $\frac{1}{2} \times \left(\frac{35}{2}\right)^2 > -\frac{43}{2}$