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(1) 解と係数の関係より． $\alpha+\beta+\gamma=-6$， $\alpha\beta+\beta\gamma+\gamma\alpha=12$， $\alpha\beta\gamma=15$．

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}=\frac{\beta^2\gamma^2+\gamma^2\alpha^2+\alpha^2\beta^2}{\alpha^2\beta^2\gamma^2}=\frac{(\alpha\beta+\beta\gamma+\gamma\alpha)^2-2\alpha\beta\gamma(\alpha+\beta+\gamma)}{(\alpha\beta\gamma)^2}=\frac{12^2-2\cdot15\cdot(-6)}{15^2}=\left(\frac{6}{5}\right)^2$$

(2) $$\sum_{k=1}^{15}\frac{1}{k(k+1)}=\sum_{k=1}^{15}\left(\frac{1}{k}-\frac{1}{k+1}\right)=\left(\frac{1}{1}-\cancel{\frac{1}{2}}\right)+\left(\cancel{\frac{1}{2}}-\cancel{\frac{1}{3}}\right)+\cdots+\left(\cancel{\frac{1}{15}}-\frac{1}{16}\right)=\frac{15}{16}$$

$$\sum_{k=1}^{15}\frac{3k+4}{k(k+1)(k+2)}=\sum_{k=1}^{15}\frac{3(k+2)-2}{k(k+1)(k+2)}=3\sum_{k=1}^{15}\frac{1}{k(k+1)}-2\sum_{k=1}^{15}\left\{\frac{1}{k(k+1)}-\frac{1}{(k+1)(k+2)}\right\}\times\frac{1}{2}$$

$$=3\times\frac{15}{16}-\frac{1}{2}+\frac{1}{16\cdot17}=\frac{315}{136}$$

(3) $$\frac{{}_5C_1\times3\times1^4}{3^5}=\frac{5}{81}$$

(4) $\log_{10}7^{53}=53\times0.845=44.785$ $\quad\therefore\quad 7^{53}=10^{44.785}=10^{0.785}\cdot10^{44}$ $\qquad$ **45桁**

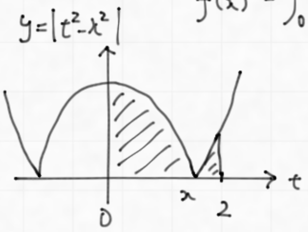
$7^1\equiv7\pmod{10}$ 以下全て10を法とする

$7^2\equiv9$ ， $7^3\equiv3$ ， $7^4\equiv1$

$$7^{53}=7^{4\times13+1}=(7^4)^{13}\cdot7\equiv1^{13}\cdot7\equiv\mathbf{7}$$

(5) $0\leqq x\leqq2$ のとき

$$f(x)=\int_0^2|t^2-x^2|dt+\frac{1}{3}=\int_0^x|t^2-x^2|dt+\int_x^2|t^2-x^2|dx+\frac{1}{3}$$



$$=\int_0^x-(t^2-x^2)dt+\int_x^2(t^2-x^2)dt+\frac{1}{3}$$

$$=\int_x^0t^2-x^2dt+\int_x^2t^2-x^2dt+\frac{1}{3}=\left[\frac{1}{3}t^3-x^2t\right]_x^0+\left[\frac{1}{3}t^3-x^2t\right]_x^2+\frac{1}{3}$$

$$=0+\frac{8}{3}-2x^2-2\left(\frac{1}{3}x^3-x^3\right)+\frac{1}{3}=\frac{4}{3}x^3-2x^2+3$$

$$\int_0^2\frac{4}{3}x^3-2x^2+3\,dx=\left[\frac{1}{3}x^4-\frac{2}{3}x^3+3x\right]_0^2=\frac{16}{3}-\frac{16}{3}+6=\mathbf{6}$$

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(1) $f'(x) = 4x^3 + 3a_3x^2 + 2a_2x + a_1$

$$Q(x) = \frac{1}{16}(4x + a_3)$$

$$R(x) = \frac{1}{16}\left\{(8a_2 - 3a_3^2)x^2 + 2(6a_1 - a_2a_3)x + (16a_0 - a_1a_3)\right\}$$

$$\begin{array}{r|lllll}
 & \frac{1}{4} & \frac{1}{16}a_3 & & & \\
4\ \ 3a_3\ \ 2a_2\ \ a_1 & 1 & a_3 & a_2 & a_1 & a_0 \\
 & 1 & \frac{3}{4}a_3 & \frac{1}{2}a_2 & \frac{1}{4}a_1 & \\
\hline
 & & \frac{1}{4}a_3 & \frac{1}{2}a_2 & \frac{3}{4}a_1 & a_0 \\
 & & \frac{1}{4}a_3 & \frac{3}{16}a_3^2 & \frac{2}{16}a_2a_3 & \frac{1}{16}a_1a_3 \\
\hline
 & & & \frac{1}{2}a_2 - \frac{3}{16}a_3^2 & \frac{3}{4}a_1 - \frac{1}{8}a_2a_3 & a_0 - \frac{1}{16}a_1a_3
\end{array}$$

(2) 解と係数の関係

$$\alpha + \beta + \gamma = -\frac{3}{4}a_3, \quad \alpha\beta + \beta\gamma + \gamma\alpha = \frac{1}{2}a_2, \quad \alpha\beta\gamma = -\frac{1}{4}a_1$$

$$a_1 = -4\alpha\beta\gamma, \quad a_2 = 2(\alpha\beta + \beta\gamma + \gamma\alpha), \quad a_3 = -\frac{4}{3}(\alpha + \beta + \gamma)$$

(3) $$\frac{f(\beta) - f(\alpha)}{\beta - \alpha} = \frac{R(\beta) - R(\alpha)}{\beta - \alpha}$$

$$= \frac{(8a_2 - 3a_3^2)(\beta + \alpha)\cancel{(\beta - \alpha)} + 2(6a_1 - a_2a_3)\cancel{(\beta - \alpha)}}{\cancel{\beta - \alpha}} \times \frac{1}{16} = (*)$$

$$8a_2 - 3a_3^2 = 16(\alpha\beta + \beta\gamma + \gamma\alpha) - 3\cdot\frac{16}{9}(\alpha + \beta + \gamma)^2 = 16\left((\alpha\beta + \beta\gamma + \gamma\alpha) - \frac{1}{3}(\alpha + \beta + \gamma)^2\right)$$

$$= -\frac{16}{3}(\alpha^2 + \beta^2 + \gamma^2 - \alpha\beta - \beta\gamma - \gamma\alpha)$$

$$2(6a_1 - a_2a_3) = 2\left(-24\alpha\beta\gamma + \frac{8}{3}(\alpha\beta + \beta\gamma + \gamma\alpha)(\alpha + \beta + \gamma)\right) = \frac{8}{3}(\alpha^2\beta + \alpha\beta^2 + \beta^2\gamma + \beta\gamma^2 + \gamma\alpha^2 + \gamma^2\alpha - 6\alpha\beta\gamma)$$

だから $$(*) = \frac{1}{3}(2\beta^2\gamma + 2\gamma\alpha^2 + \alpha\beta^2 + \alpha^2\beta - \alpha^3 - \beta^3 - 4\alpha\beta\gamma)$$

$$= \frac{1}{3}\left\{2\gamma(\beta - \alpha)^2 + \alpha(\beta + \alpha)(\beta - \alpha) - \beta(\beta + \alpha)(\beta - \alpha)\right\} = \frac{1}{3}(\beta - \alpha)^2(2\gamma - \alpha - \beta)$$

(4) α, β, γ が等差数列だから 等差中項の公式より $2\beta = \alpha + \gamma$

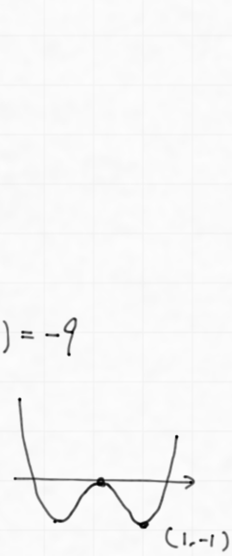
BC の傾きは $$\frac{f(\gamma) - f(\beta)}{\gamma - \beta} = \frac{1}{3}(\gamma - \beta)^2(2\alpha - \beta - \gamma)$$

ABとBCが直交するので $$\frac{1}{3}(\beta - \alpha)^2(2\gamma - \alpha - \beta) \times \frac{1}{3}(\gamma - \beta)^2(2\alpha - \beta - \gamma) = -1$$

$\beta = \frac{1}{2}\alpha + \frac{1}{2}\gamma$ を代入 $$\left(\frac{1}{2}\gamma - \frac{1}{2}\alpha\right)^2\left(\frac{3}{2}\gamma - \frac{3}{2}\alpha\right) \times \left(\frac{1}{2}\gamma - \frac{1}{2}\alpha\right)^2\left(\frac{3}{2}\alpha - \frac{3}{2}\gamma\right) = -9$$

$$\frac{\cancel{3^2}}{2^6}(\gamma - \alpha)^6 = \cancel{9} \qquad \gamma - \alpha = 2$$

$$\beta - \alpha = \frac{1}{2}\gamma - \frac{1}{2}\alpha = 1$$



(5) y軸対称 → $f(x)$ は偶関数 → $f(x) = x^4 + a_2x^2 + a_0$, $\alpha = -\gamma$ $\qquad \alpha = -1,\ \gamma = 1.$ $\beta = 0$

$a_2 = -2$, $f(1) = 1 - 2 + a_0 = -1 + a_0$, $f(0) = a_0$, $f(-1) = -1 + a_0$ だから.

$(0, a_0), (1, -1 + a_0), (-1, -1 + a_0)$ を通る 2次関数 $y = -x^2 + a_0$

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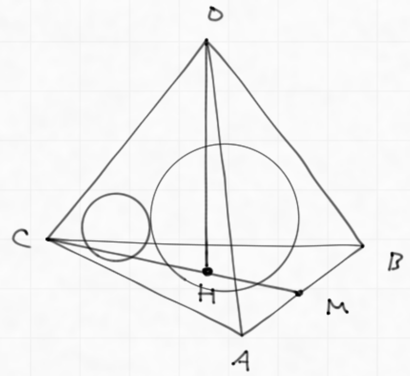
(1) 右下図より.



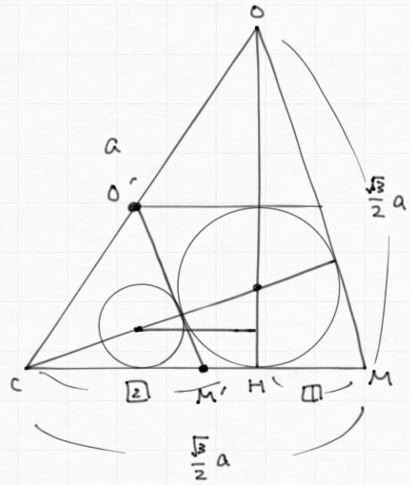
$$MH = \frac{1}{3}CM = \frac{\sqrt{3}}{2}\times\frac{1}{3}a = \frac{\sqrt{3}}{6}a.$$

$$OH = \sqrt{\left(\frac{\sqrt{3}}{2}a\right)^2 - \left(\frac{\sqrt{3}}{6}a\right)^2} = a\sqrt{\frac{3}{4}-\frac{1}{12}} = \frac{\sqrt{6}}{3}a$$

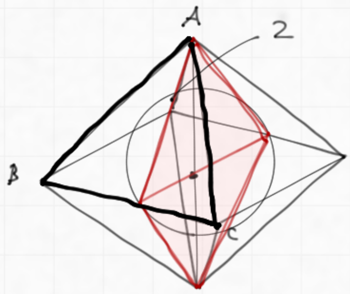
Xの半径 $\frac{1}{4}OH = \frac{\sqrt{6}}{12}a$

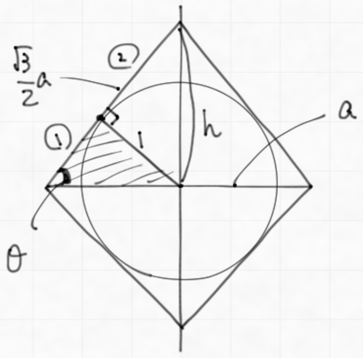
$\triangle COM \backsim \triangle CO'M'$ でその比は $2:1$ だから

Yの半径は $\frac{1}{2}\times\frac{\sqrt{6}}{12}a = \frac{\sqrt{6}}{24}a$



(2) 右下図のような断面を考える, 辺の長さを a として.





左図の斜線部三角形で

三平方の定理を用いて

$$1^2 + \left\{\frac{1}{3}\times\left(\frac{\sqrt{3}}{2}a\right)\right\}^2 = \left(\frac{a}{2}\right)^2$$

$$1 + \frac{1}{12}a^2 = \frac{1}{4}a^2$$

$$a = \sqrt{6}$$

図中の $h = \sqrt{1^2 + \left(\frac{\sqrt{3}}{2}\times\sqrt{6}\times\frac{2}{3}\right)^2} = \sqrt{1+2} = \sqrt{3}$

体積Vは $V = \sqrt{6}^2 \times \sqrt{3} \times \frac{1}{3} \times 2 = 4\sqrt{3}$

図中の θ について. $\sin\theta = \dfrac{\sqrt{3}}{\frac{\sqrt{3}}{2}\times\sqrt{6}} = \dfrac{2}{\sqrt{6}} = \dfrac{\sqrt{6}}{3}$ $\cos\theta = \dfrac{\frac{\sqrt{6}}{2}}{\frac{\sqrt{3}}{2}\times\sqrt{6}} = \dfrac{\sqrt{3}}{3}$

$$\sin 2\theta = 2\sin\theta\cos\theta = 2\times\frac{\sqrt{6}}{3}\times\frac{\sqrt{3}}{3} = \frac{2\sqrt{2}}{3}$$

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IV

A Z B U V E T
A

$\dfrac{8!}{2!} = 8\cdot7\cdot6\cdot5\cdot4\cdot3 = 20160$ 通り

AZABU . V , E , T を並べる　$4! = 24$ 通り

○×○×○×○×　${}_4C_2 \times 6! = 6 \times 6\cdot5\cdot4\cdot3\cdot2 = 4320$ 通り

Z , B , U , V , E , T を並べる　$6!$

すきまにAを入れる　${}_7C_2$　$\therefore\ 6! \times {}_7C_2 = 720 \times 21 = 15120$ 通り