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(1) $(x+y)^2 = 3\sqrt{2}+2$

$(x-y)^2 = 3\sqrt{2}-2$

$2(x^2+y^2) = 6\sqrt{2}$　　　$x^2+y^2 = 3\sqrt{2}$

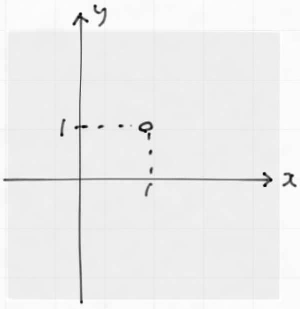
$4xy = 4$　　　$xy = 1$

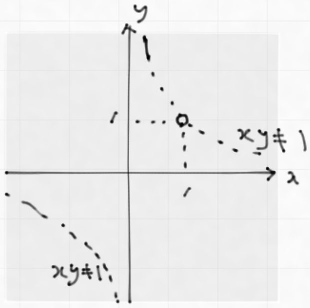
$x^2+y^2-xy = \mathbf{3\sqrt{2}-1}$

(2) $3600 = 6^2\times 10^2 = 2^4\cdot 3^2\cdot 5^2$　　　60 の約数は 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

$n = 1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 10^2, 12^2, 15^2, 20^2, 30^2, 60^2$ の **12** コ

(3)





①

(4) 元の平均 $(3+6+7+8)\div 4 = 6$

5個の平均 $(3+6+7+8+x)\div 5 = 6-1$　　　$x = 1$

分散 $\dfrac{1^2+3^2+6^2+7^2+8^2}{5} - 5^2 = \dfrac{159}{5} - 25 = \dfrac{34}{5} = \mathbf{6.8}$

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2. (1) 相加相乗

$$4x^2+y^2 \geqq 2\sqrt{4x^2 \cdot y^2} = 4xy = \mathbf{20}$$

(2)

$$2^x = X > 0$$

$$y = 4X^2 - 2X + 1 = 4\left(X-\frac{1}{4}\right)^2 + \frac{3}{4}$$

$X=\frac{1}{4}$ のとき y は最小値 $\mathbf{\frac{3}{4}}$ をとる

(3)

$$|\vec{a}-2\vec{b}|^2 = |\vec{a}|^2 - 4\vec{a}\cdot\vec{b} + 4|\vec{b}|^2 = 17 - 4\vec{a}\cdot\vec{b}$$

$\vec{a}\cdot\vec{b} = 1\cdot 2\cdot \cos\theta \quad (0 \leqq \theta \leqq 2\pi)$ と考えられるので.

$$|\vec{a}-2\vec{b}|^2 = 17 - 8\cos\theta$$

$$17-8 \leqq |\vec{a}-2\vec{b}|^2 \leqq 17+8$$

$$\mathbf{3} \leqq |\vec{a}-2\vec{b}| \leqq \mathbf{5}$$

(4)

$$a_n = a_1 + \sum_{k=1}^{n-1}(4k+2) = 2 + \frac{6+4n-2}{2}\times(n-1) = 2n^2$$

$2n^2 > 1000 \quad n^2 > 500 > 22^2$ 第 **23** 項

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(1) $4x+3y=25$ 　　x軸との交点は $\frac{25}{4}$, y軸との交点は $\frac{25}{3}$

$$\sqrt{\left(\frac{25}{4}\right)^2+\left(\frac{25}{3}\right)^2}=25\sqrt{\frac{1}{4^2}+\frac{1}{3^2}}=25\sqrt{\frac{5^2}{4^2\cdot 3^2}}=\frac{5\cdot 25}{4\cdot 3}=\frac{125}{12}$$

(2) ABの中点 $\left(\frac{4+(-3)}{2},\ \frac{3+4}{2}\right)=\left(\frac{1}{2},\frac{7}{2}\right)$

ABの傾き $\frac{4-3}{-3-4}=-\frac{1}{7}$

ABの垂直二等分線 $y=7\left(x-\frac{1}{2}\right)+\frac{7}{2}$ 　　$7x-y=0$

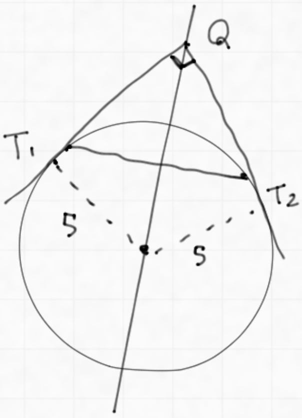
(3) $7x-y=0$ と $x^2+y^2=25$ の交点は $50x^2=25$ 　$x=\pm\frac{1}{\sqrt{2}}$.

図より. $(x,y)=\left(-\frac{1}{\sqrt{2}},-\frac{7}{\sqrt{2}}\right)$ これを P_0 とすると

$\overrightarrow{P_0A}=\left(4+\frac{1}{\sqrt{2}},3+\frac{7}{\sqrt{2}}\right)$, $\overrightarrow{P_0B}=\left(-3+\frac{1}{\sqrt{2}},4+\frac{7}{\sqrt{2}}\right)$

$$\triangle AP_0B=\frac{1}{2}\left|\left(4+\frac{1}{\sqrt{2}}\right)\left(4+\frac{7}{\sqrt{2}}\right)-\left(3+\frac{7}{\sqrt{2}}\right)\left(-3+\frac{1}{\sqrt{2}}\right)\right|=\frac{1}{2}\left|16+\frac{28}{\sqrt{2}}+\frac{4}{\sqrt{2}}+\cancel{\frac{7}{2}}+9-\frac{3}{\sqrt{2}}+\frac{21}{\sqrt{2}}-\cancel{\frac{7}{2}}\right|$$

$$=\frac{1}{2}\left|25+25\sqrt{2}\right|=\frac{25}{2}(\sqrt{2}+1)$$



Q $(t,7t)$. 　　$OQ=\sqrt{t^2+(7t)^2}=5t\sqrt{2}=5\sqrt{2}$

$t=1$, 　Q $(1,7)$

$\frac{5}{OQ}\leqq\sin 15^\circ$ 　　$\frac{\cancel{5}}{\cancel{5}t\sqrt{2}}\leqq\frac{\sqrt{6}-\sqrt{2}}{4}$

$$t\geqq\frac{\cancel{4}\ 2\sqrt{2}}{\cancel{4}2(\sqrt{6}-\sqrt{2})}=\frac{\cancel{2\sqrt{2}}(\sqrt{6}+\sqrt{2})}{\cancel{4}\ 2}=\sqrt{3}+1$$

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(1) $f(x) = cx^2+dx+e$ とする

$$x^2f'(x) - f(x) = x^2(2cx+d) - cx^2-dx-e = 2cx^2+(d-c)x^2-dx-e$$

これが x^3+ax^2+bx と一致する.

$$2c=1,\ d-c=a,\ -d=b,\ -e=0$$

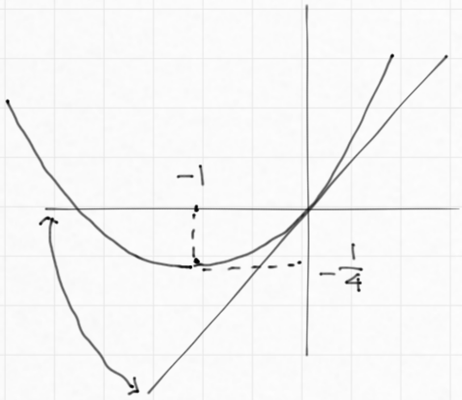
$$c=\frac{1}{2},\ e=0.\ d=a+\frac{1}{2},\ b=-a-\frac{1}{2},\ e=0.$$

$$f(x)=\frac{1}{2}x^2+\left(a+\frac{1}{2}\right)x$$

(2) $f(x)=\frac{1}{2}x(x+2a+1)$ 　　$f(x)=0$ となるのは 0, $-2a-1$.

$$S=\frac{\frac{1}{2}}{6}\{0-(-2a-1)\}^3=\frac{1}{12}(2a+1)^3=\frac{2}{3}$$

$$(2a+1)^3=8 \qquad 2a+1=2 \qquad a=\frac{1}{2}$$



(3) $f(x)=\frac{1}{2}x^2+x=\frac{1}{2}(x+1)^2-\frac{1}{4}$

$$f'(x)=x+1 \qquad f'(0)=1$$

右図より　$0<m<1$

$$\int_\alpha^0 mx-f(x)\,dx=\frac{1}{3}=\frac{\frac{1}{2}}{6}(0-\alpha)^3=-\frac{1}{12}\alpha^3$$

$$\alpha^3=-4 \qquad \alpha=-\sqrt[3]{4}$$

$$f(\alpha)=\frac{1}{2}\alpha^2+\alpha=m\cdot\alpha \qquad m=\frac{1}{2}\alpha+1=1+2^{-1}\cdot(-2^{\frac{2}{3}})=1-2^{-\frac{1}{3}}$$

$$=1-\frac{2^{\frac{2}{3}}}{2}=1-\frac{\sqrt[3]{4}}{2}$$

(4) $\int_{-1}^{p} x^2(x+1)\,dx=\left[\frac{1}{4}x^4+\frac{1}{3}x^3\right]_{-1}^{p}=\frac{27}{4}$

$$\frac{1}{4}p^4+\frac{1}{3}p^3-\frac{1}{4}+\frac{1}{3}=\frac{27}{4}$$

$$3p^4+4p^3-80=0 \qquad p=2$$