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I

(1) $x^3+y^3+z^3=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)+3xyz$

$=6\{(x+y+z)^2-3(xy+yz+zx)\}+9 = 6(36-12)+9=\mathbf{153}$

(2) $x+22=11n^2$ となればよい.

$n=10 \quad x=1100-22=1078$

$n=9 \quad x=891-22=\mathbf{869}$

(3) $\log_{10}18^{35}=35(\log 3^2+\log 2)=35(0.4771\times 2+0.3010)=43.974$ **44桁**

(4) $\dfrac{104}{51}=\dfrac{2^3\cdot 13}{3\cdot 17}$, $\dfrac{156}{85}=\dfrac{2^2\cdot 3\cdot 13}{5\cdot 17}$ $\dfrac{3\cdot 5\cdot 17}{2^2\cdot 13}=\mathbf{\dfrac{255}{52}}$

(5) (X,Y) とおく.

距離が等しいので $\dfrac{|X-2Y+1|}{\sqrt{1^2+(-2)^2}}=\dfrac{|2X+Y-3|}{\sqrt{2^2+1^2}} \iff X-2Y+1=\pm(2X+Y-3)$

$X+3Y-4=0$ または $3X-Y-2=0$

$\therefore\ x+\mathbf{3}y\mathbf{-4}=0$, $\mathbf{3}x-y-\mathbf{2}=0$

(6) $\dfrac{a_{n+1}}{6^{n+1}}=\dfrac{a_n}{6^n}+\left(\dfrac{1}{2}\right)^{n+1}$ $\dfrac{a_n}{6^n}=\dfrac{a_1}{6^1}+\displaystyle\sum_{k=1}^{n-1}\left(\dfrac{1}{2}\right)^{k+1}=\dfrac{1}{2}+\dfrac{1}{4}\times\dfrac{1-\left(\frac{1}{2}\right)^{n-1}}{1-\frac{1}{2}}=1-\left(\dfrac{1}{2}\right)^n$

$a_n=6^n-3^n=\mathbf{3^n(2^n-1)}$

(7) $n^{24}=\dfrac{1}{8}$ $n^8=\dfrac{1}{2}$ **8枚**

(8) $\dfrac{3}{1+2i}=\dfrac{3(1-2i)}{1+4}=\dfrac{3}{5}-\dfrac{6}{5}i$ 共役複素数も解 $\dfrac{3}{5}+\dfrac{6}{5}i$

$\dfrac{3}{5}-\dfrac{6}{5}i+\dfrac{3}{5}+\dfrac{6}{5}i=\dfrac{6}{5}$. $\left(\dfrac{3}{5}-\dfrac{6}{5}i\right)\left(\dfrac{3}{5}+\dfrac{6}{5}i\right)=\dfrac{9}{25}+\dfrac{36}{25}=\dfrac{9}{5}$

解と係数の関係より $a=\mathbf{-\dfrac{6}{5}}$. $b=\mathbf{\dfrac{9}{5}}$

(9) $\dfrac{\cos^2\theta}{1+2\sin\theta\cos\theta}=\dfrac{1}{\frac{1}{\cos^2\theta}+2\tan\theta}=\dfrac{1}{1+\tan^2\theta+2\tan\theta}=\dfrac{1}{3+2\sqrt{2}}=\mathbf{3-2\sqrt{2}}$

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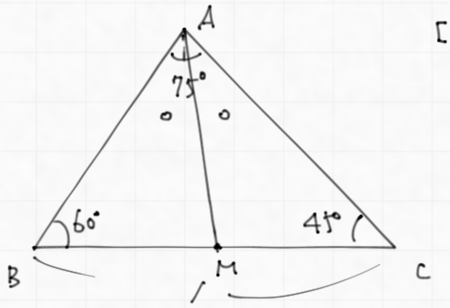
Ⅱ



[1] $180 - 75 = 105$

$105 \times \dfrac{4}{4+3} = 60.$　$105 - \dfrac{3}{4+3} = 45°$

(1) $\sin 75° = \sin(45° + 30°) = \dfrac{\sqrt{2} + \sqrt{6}}{4}$

(2) $2R = \dfrac{1}{\sin 75°}$　$R = \dfrac{4}{2(\sqrt{2} + \sqrt{6})} = \dfrac{\sqrt{6} - \sqrt{2}}{2}$

(3) 正弦定理より

$2R = \dfrac{AB}{\sin 45°}$　$AB = (\sqrt{6} - \sqrt{2}) \times \dfrac{1}{\sqrt{2}} = \sqrt{3} - 1$

$2R = \dfrac{AC}{\sin 60°}$　$AC = (\sqrt{6} - \sqrt{2}) \times \dfrac{\sqrt{3}}{2} = \dfrac{3\sqrt{2} - \sqrt{6}}{2}$

(4) $AM = \sqrt{BM^2 + AB^2 - 2AB \cdot BM \cdot \cos 60°} = \sqrt{\dfrac{1}{4} + 4 - 2\sqrt{3} - (\sqrt{3} - 1) \cdot \dfrac{1}{2}} = \dfrac{\sqrt{19 - 10\sqrt{3}}}{2}$

[2] $y = x^2 - 2x + 1$, $y = mx$

連立して $x^2 - 2x + 1 = mx$ …①

判別式 $D = (-2-m)^2 - 4 > 0$　$m^2 + 4m > 0$　$m(m+4) > 0$.

$m < -4$, $m > 0$ …②

① の解を α, β として. 解と係数の関係より $\alpha + \beta = 2 + m$. $\alpha\beta = 1$

P.Q の座標は $(\alpha, m\alpha)$, $(\beta, m\beta)$

中点を (X, Y) として　$X = \dfrac{\alpha + \beta}{2} = \dfrac{2+m}{2}$, $Y = \dfrac{m\alpha + m\beta}{2} = m + \dfrac{1}{2}m^2$

$m = 2X - 2$ を代入　$Y = 2X - 2 + \dfrac{1}{2}(2X-2)^2 = 2X^2 - 2X$

$y = 2x(x-1)$

$m = 2X - 2$ を ② に代入　$2X - 2 < -4$, $2X - 2 > 0$ $\Leftrightarrow$ $X < -1$, $X > 1$

$x < -1$, $x > 1$

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III

(1) $I(p, q, r) = \int_{-1}^{1} (x^3 + px^2 + qx + r)^2 dx$

$= 2\int_0^1 x^6 + p^2x^4 + q^2x^2 + r^2 + 2qx^4 + 2prx^2 \, dx$

$= 2\int_0^1 (px^2+r)^2 + x^6 + q^2x^2 + 2qx^4 \, dx$

$= 2\int_0^1 (px^2+r)^2 dx + 2\left[\frac{1}{7}x^7 + \frac{1}{3}q^2x^3 + \frac{2}{5}qx^5\right]_0^1$

$= 2\int_0^1 (px^2+r)^2 dx + 2\left(\frac{1}{7} + \frac{1}{3}q^2 + \frac{2}{5}q\right)$

$= 2\int_0^1 (px^2+r)^2 dx + 2\left\{\frac{1}{3}\left(q+\frac{3}{5}\right)^2 + \frac{4}{175}\right\}$

$\therefore \frac{1}{2}I(p, q, r) = \int_0^1 (px^2+r)^2 dx + \frac{1}{3}\left(q+\frac{3}{5}\right)^2 + \frac{4}{175}$

px^2+r が常に 0. $q+\frac{3}{5}=0$ のとき最小. $p=r=0$, $q=-\frac{3}{5}$ 最小値 $\frac{8}{175}$

$\int_{-1}^{1} \left(x^3 - \frac{3}{5}x\right)(ax^2+bx+c) dx = 2\int_0^1 bx^4 - \frac{3}{5}bx^2 \, dx$

$= 2\left[\frac{1}{5}bx^5 - \frac{1}{5}bx^3\right]_0^1 = 0$

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IV

[1] $p_n = \left(\frac{1}{6}\right)^n \left(\frac{5}{6}\right)^{100-n} \times {}_{100}C_n$

$$\frac{p_{n+1}}{p_n} - 1 = \frac{\left(\frac{1}{6}\right)^{n+1}\left(\frac{5}{6}\right)^{99-n} \times {}_{100}C_{n+1}}{\left(\frac{1}{6}\right)^{n}\left(\frac{5}{6}\right)^{100-n} \times {}_{100}C_{n}} - 1 = \frac{\cancel{n!}\,(100-n)\cancel{!}\cdot\cancel{100!}}{5\cdot\cancel{100!}\,(n+1)\cancel{!}\,\cancel{(99-n)!}} - 1$$

$$= \frac{100-n}{5(n+1)} - 1 = \frac{\mathbf{95-6n}}{\mathbf{5}(n+1)}$$

$p_n < p_{n+1}$ より $\frac{p_{n+1}}{p_n} > 1 \Leftrightarrow \frac{p_{n+1}}{p_n} - 1 > 0$ となるので

$95-6n>0$ のとき $p_n < p_{n+1}$ が成り立つ. $n < \frac{95}{6} = 15.\cdots$ $n = \mathbf{15}$

$p_n > p_{n+1}$ が成り立つ最小の n は $n = \mathbf{16}$

$p_1 < p_2 < \cdots < p_{15} < p_{16} > p_{17} > \cdots$ p_{16} が最大. $n = \mathbf{16}$

[2] $$q_n = \frac{{}_{15}C_1 \times {}_nC_1}{{}_{15+n}C_2} = \frac{2\cdot 15\cdot n}{(15+n)(14+n)} = \frac{\mathbf{30}\,n}{(\mathbf{14}+n)(\mathbf{15}+n)}$$

$$\frac{q_{n+1}}{q_n} = \frac{\dfrac{30(n+1)}{\cancel{(15+n)}(16+n)}}{\dfrac{30n}{(14+n)\cancel{(15+n)}}} = \frac{(14+n)\cdot\cancel{30}(n+1)}{(16+n)\cdot\cancel{30}n} > 1$$

$14 + \cancel{n^2} + 15n > \cancel{n^2} + 16n$

$n < 14.$

$q_1 < q_2 < \cdots < q_{13} < q_{14} = q_{15} > q_{16} > \cdots$ $n = \mathbf{14}$ または $\mathbf{15}$ のとき最大

$$q_{14} = q_{15} = \frac{\overset{15}{\cancel{30}} \times \cancel{14}}{\underset{2}{\cancel{28}} \times 29} = \frac{\mathbf{15}}{\mathbf{29}}$$