

# 2020 藤田医大(後)

$$(1) x = 46656 = 2^6 \cdot 3^6 \quad e^{2\log_2 6 \cdot 6} = 2^{12} \cdot 3^{12} = y^3 \quad y = 2^4 \cdot 3^4 = 6^4 = 1296$$

$$(2) \text{下記の } F \text{ と } G \text{ は} \quad P(X \cap F) = \frac{5}{7} \times \frac{2}{100} = \frac{10}{700} \quad P(Y \cap F) = \frac{2}{7} \times \frac{1}{100} = \frac{2}{700}$$

$$P_F(x) = \frac{P(F \cap X)}{P(F)} = \frac{\frac{10}{700}}{\frac{10}{700} + \frac{2}{700}} = \frac{5}{6}$$

$$(3) \lim_{x \rightarrow 3} (a\sqrt{x+13} - b) = 4a - b = 0 \quad b = 4a.$$

$$\lim_{x \rightarrow 3} \frac{a(\sqrt{x+13} - 4)}{x-3} = \lim_{x \rightarrow 3} \frac{a(\sqrt{x+13} - 4)}{(x-3)(\sqrt{x+13} + 4)} = \frac{a}{8} = \frac{1}{4} \quad a=2, b=8$$

$$(4) Z = 2\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 2\left(\cos(-\frac{\pi}{3}) + i\sin(-\frac{\pi}{3})\right)$$

$$\begin{aligned} Z^7 + aZ^5 - b &= 2^7 \left(\cos(-\frac{7}{3}\pi) + i\sin(-\frac{7}{3}\pi)\right) + a \cdot 2^5 \left(\cos(-\frac{5}{3}\pi) + i\sin(-\frac{5}{3}\pi)\right) - b \\ &= 2^7 \left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) + 2^5 a \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) + b = 2^6 + 2^4 a + b + (-2^6 \sqrt{3} + 2^4 \sqrt{3} a)i = 0 \end{aligned}$$

$$64 + 16a + b = 0 \Rightarrow -64\sqrt{3} + 16\sqrt{3}a = 0 \quad \therefore a = 4, b = 128$$

$$\begin{aligned} (5) f'(x) &= \frac{\cos x (\sin x + 2\cos x) - \sin x (\cos x - 2\sin x)}{(\sin x + 2\cos x)^2} = \frac{2}{(\sin x + 2\cos x)^2} = \frac{2}{\sin^2 x + 4\sin x \cos x + 4\cos^2 x} \\ &= \frac{2}{\frac{1-\cos 2x}{2} + 2\sin 2x + 4 \cdot \frac{1+\cos 2x}{2}} = \frac{4}{3\cos 2x + 4\sin 2x + 5} \end{aligned}$$

$$f'(\frac{\pi}{4}) = \frac{2}{(\frac{1}{\sqrt{2}} + \frac{2}{\sqrt{2}})^2} = \frac{4}{9}$$

$$\frac{1}{4} f'(\frac{\pi}{8}) = \frac{1}{3 \cdot \cos \frac{\pi}{4} + 4 \sin \frac{\pi}{4} + 5} = \frac{\sqrt{2}}{7 + 5\sqrt{2}} = \sqrt{2}(5\sqrt{2} - 7) = 10 - 7\sqrt{2}$$

(6)  $a \neq 0$  のとき  $b^2 - 4a > 0$  のとき 異なる3実数解をもつ

$$b^2 > 4a \quad b=1 \text{ のとき} \quad a < \frac{1}{4} \quad a \text{ は負値 (左)}$$

$$b=2 \quad " \quad a < 1 \quad "$$

$$b=3 \quad " \quad a < \frac{9}{4} \quad a=1, 2,$$

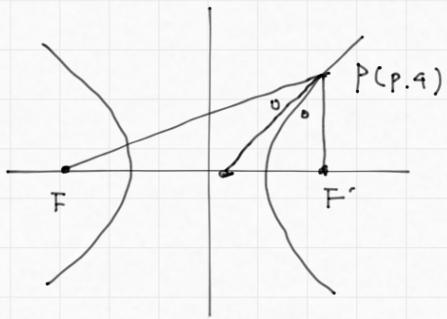
$$b=4 \quad " \quad a < 4 \quad a=1, 2, 3$$

$$b=5 \quad " \quad a < \frac{25}{4} \quad a=1, 2, 3, 4, 5, 6$$

$$b=6 \quad " \quad a < 9 \quad a=1, 2, 3, 4, 5, 6, 7, 8$$

$$\therefore \frac{19}{6 \times 8} = \frac{19}{48}$$

(7)



$$\sqrt{16+q} = 5 \text{, たゞか } F(-5,0), F'(5,0)$$

$$P \text{ は双曲線上にあるので } \frac{P^2}{16} - \frac{q^2}{9} = 1 \quad \therefore p = \frac{20}{3}$$

$$P \text{ における } F \text{ と } F' \text{ の距離は } \frac{x}{16} \times \frac{20}{3} - \frac{y}{9} \times 4 = 1$$

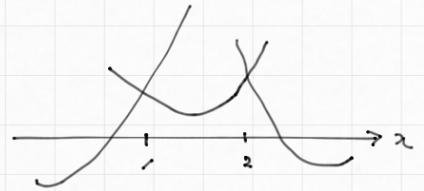
$$y=0 \text{ のとき } x = \frac{12}{5} \quad \therefore \left( \frac{12}{5}, 0 \right)$$

$$(8) \int_0^{\frac{\pi}{4}} \frac{\frac{d}{dx}(\cos x)}{\cos^7 x} dx = \left[ -\frac{1}{6} \cos^{-6} x \right]_0^{\frac{\pi}{4}} = \frac{7}{6}$$

$$(9) f(x) = \left( x + \frac{a}{2} \right)^2 - \frac{a^2}{4} + 2a - 3$$

$$(i) \left[ -\frac{a}{2} < x < 2 \right] \text{ で } -\frac{a^2}{4} + 2a - 3 > 0$$

$$(ii) (i) \text{ 以外のとき. } f(1) \geq 0 \text{ かつ } f(2) \geq 0$$

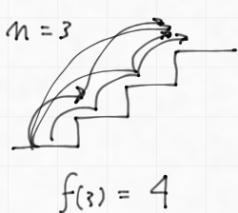
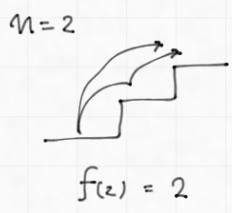
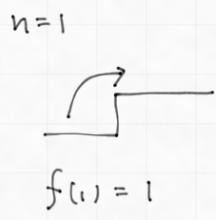


$$(i) -4 < a < -2, \quad 2 < a < 6 \quad \dots \text{ 解なし}$$

$$(ii) f(1) = 3a - 2 \geq 0, \quad f(2) = 4a + 1 \geq 0, \quad a \leq -4, \quad a \geq -2 \quad \therefore a \geq \frac{2}{3}$$

$$(i) (ii) \text{ より } a \geq \frac{2}{3}$$

(10)  $n$  個の階段  $\frac{1}{n}$  の  $f_n$  を  $f(n)$  とする.



$n+3$  のとき

$$\begin{array}{ll} \text{最初の} & \text{最後の} \\ \text{段} & \text{段} \\ \Rightarrow & \Rightarrow \\ \Rightarrow & \Rightarrow \\ \Rightarrow & \Rightarrow \end{array} \begin{array}{ll} n+2 & n+1 \\ \text{段} & \text{段} \\ \Rightarrow & \Rightarrow \\ \Rightarrow & \Rightarrow \\ \Rightarrow & \Rightarrow \end{array} \begin{array}{l} f(n+2) \\ f(n+1) \\ f(n) \end{array}$$

$$f(n+3) = f(n+2) + f(n+1) + f(n)$$

$$f(4) = f(3) + f(2) + f(1) = 4 + 2 + 1 = 7$$

$$f(5) = 7 + 4 + 2 = 13$$

$$f(6) = 13 + 7 + 4 = 24$$

$$f(7) = 24 + 13 + 7 = 44$$

$$f(8) = 44 + 24 + 13 = 81$$

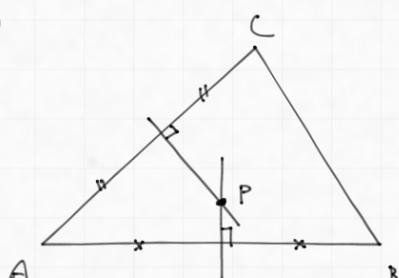
$$f(9) = 81 + 44 + 24 = 149$$

$$f(10) = 149 + 81 + 44 = 274$$

$$f(11) = 274 + 149 + 81 = 504$$

$$f(12) = 504 + 274 + 149 = 927$$

(11)



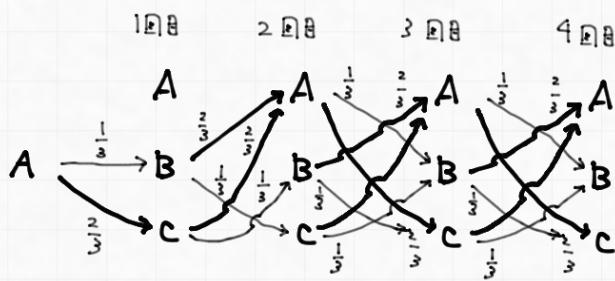
PはABの垂直二等分線上にある。

$$(\vec{AP} - \frac{1}{2}\vec{AB}) \cdot \vec{AB} = 0$$

$$\vec{AP} \cdot \vec{AB} - \frac{1}{2}|\vec{AB}|^2 = 0$$

$$\vec{AB} \cdot \vec{AP} = \frac{1}{2} \times \delta^2 = 32$$

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(1)

$$a_1 = 0$$

$$a_2 = \frac{1}{3} \times \frac{2}{3} + \frac{2}{3} \times \frac{2}{3} = \frac{6}{9} = \frac{2}{3}$$

$$a_3 = \frac{2}{3} \times \frac{1}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} = \frac{6}{27} = \frac{2}{9}$$

$$a_4 = (1 - a_3) \times \frac{2}{3} = \frac{7}{9} \times \frac{2}{3} = \frac{14}{27}$$

(2)  $n$ 回目、Aがボールをもっているとき、 $n+1$ 回目にAがボールを持つことはない。

$n$ 回目、BまたはCがボールを持つときは、Aは、Bにも、Cにも  $\frac{2}{3}$  の確率で勝つので  $\frac{2}{3}$  の確率でボールを持つ。したがって

$$a_{n+1} = (1 - a_n) \times \frac{2}{3}$$

が成り立つ。こやう

$$a_{n+1} - \frac{2}{5} = -\frac{2}{3} (a_n - \frac{2}{5})$$

が成り立つが。これは  $\{a_n - \frac{2}{5}\}$  が初項  $a_1 - \frac{2}{5}$ 、公比  $-\frac{2}{3}$  の等比数列であることを示しているので

$$a_n - \frac{2}{5} = (a_1 - \frac{2}{5}) \times (-\frac{2}{3})^{n-1}$$

$$a_n = -\frac{2}{5} \left(-\frac{2}{3}\right)^{n-1} + \frac{2}{5}$$

$b_{n+1}$  は  $\frac{1}{3}$  で。 $n$ 回目 Bがボールを握っているとき、 $n+1$ 回目に Bがボールを持つことはない。

AまたはCがボールを持つときは Bは、AにもCにも  $\frac{1}{3}$  の確率で勝つので

$$b_{n+1} = (1 - b_n) \times \frac{1}{3}$$

$$b_{n+1} - \frac{1}{4} = -\frac{1}{3} (b_n - \frac{1}{4})$$

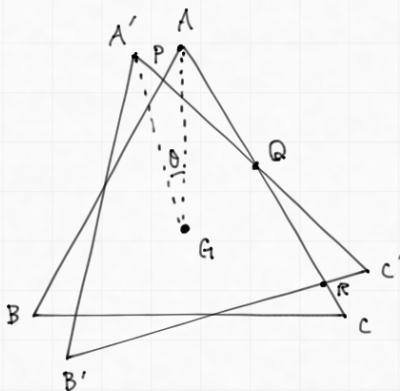
$$\text{先と同様 } b_n - \frac{1}{4} = (\frac{1}{3} - \frac{1}{4}) \left(-\frac{1}{3}\right)^{n-1} \quad \therefore b_n = -\frac{1}{4} \left(-\frac{1}{3}\right)^n + \frac{1}{4}$$

$$(3) \quad c_n = 1 - a_n - b_n = 1 + \frac{2}{5} \left(-\frac{2}{3}\right)^{n-1} - \frac{2}{5} + \frac{1}{4} \left(-\frac{1}{3}\right)^n - \frac{1}{4}$$

$$= \frac{7}{20} + \frac{2}{5} \left(-\frac{2}{3}\right)^{n-1} + \frac{1}{4} \left(-\frac{1}{3}\right)^n$$

$$\therefore \lim_{n \rightarrow \infty} c_n = \frac{7}{20}$$

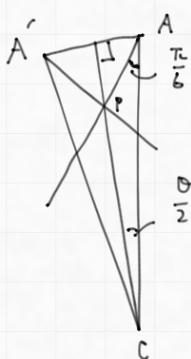
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(1) AからBCに下ろした垂線の長さは  $AB \cos \frac{\pi}{6}$  だから  $\frac{3}{2}$   
よし  $AG = \frac{3}{2} \times \frac{2}{3} = 1$   $\triangle GAA'$  は  $AG = A'G$  の  $\Rightarrow$  等辺三角形なので  
 $AA' = AG \sin \frac{\theta}{2} \times 2 = 2\sqrt{1 - \cos^2 \frac{\theta}{2}} = 2\sqrt{1 - \frac{1}{1+u^2}}$   
 $= 2\sqrt{\frac{u^2}{1+u^2}} = \frac{2u}{\sqrt{1+u^2}}$

(2)  $\triangle APG$ について  $\angle AGP = \frac{\theta}{2}$ ,  $\angle GAP = \frac{\pi}{6}$  だから  
 $\angle APG = \pi - \angle AGP - \angle GAP = \frac{5\pi}{6} - \frac{\theta}{2}$

正弦定理より



$$\frac{AG}{\sin(\frac{5\pi}{6} - \frac{\theta}{2})} = \frac{AP}{\sin \frac{\theta}{2}}$$

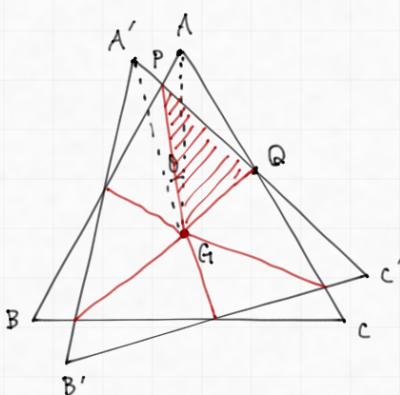
$$\Leftrightarrow AP = l(u) = 1 \times \frac{\sin \frac{\theta}{2}}{\sin(\frac{5\pi}{6} - \frac{\theta}{2})} = \frac{\sin \frac{\theta}{2}}{\sin \frac{5\pi}{6} \cos \frac{\theta}{2} - \cos \frac{5\pi}{6} \sin \frac{\theta}{2}}$$

$$= \frac{\tan \frac{\theta}{2}}{\frac{1}{2} \times 1 + \frac{\sqrt{3}}{2} \tan \frac{\theta}{2}} = \frac{2u}{1+\sqrt{3}u}$$

(3)  $\lim_{u \rightarrow \sqrt{3}} l(u) = \lim_{u \rightarrow \sqrt{3}} \frac{2u}{1+\sqrt{3}u} = \frac{\sqrt{3}}{2}$

(4) ACとAC'の交点をQとみく。

$$\angle PGQ = 2\pi \times \frac{1}{6} = \frac{\pi}{3}$$



$$\frac{l(u)}{\sin \frac{\theta}{2}} = \frac{PG}{\sin \angle GAP} \text{ より}$$

$$PG = \frac{l(u)}{\sin \frac{\theta}{2}} \times \sin \frac{\pi}{6} = \frac{l(u)}{2 \sin \frac{\pi}{2}}$$

$$\angle AQQ = \pi - \angle GAQ - \angle AGQ = \pi - \frac{\pi}{6} - (\frac{\pi}{3} - \frac{\theta}{2}) = \frac{\pi}{2} + \frac{\theta}{2}$$

$$\frac{GQ}{\sin \angle GAQ} = \frac{AG}{\sin \angle AGQ} \text{ より}$$

$$GQ = \frac{1}{\cos \frac{\theta}{2}} \times \sin \frac{\pi}{6} = \frac{1}{2 \cos \frac{\theta}{2}}$$

$$\Delta PGQ = \frac{1}{2} PG \cdot GQ \cdot \sin \frac{\pi}{3} = \frac{1}{2} \cdot \frac{l(u)}{2 \sin \frac{\pi}{2}} \cdot \frac{1}{2 \cos \frac{\theta}{2}} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3} l(u)}{16 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}$$

$$S(u) = \Delta PGQ \times 6 = \frac{3\sqrt{3}}{8} l(u) \cdot \frac{1}{\sin \frac{\theta}{2} \cos \frac{\theta}{2}} = \frac{3\sqrt{3}}{8} \times \frac{2u}{1+\sqrt{3}u} \times \frac{\frac{1}{\cos \frac{\theta}{2}}}{\tan \frac{\theta}{2}} = \frac{3\sqrt{3} \sqrt{1+u^2}}{4(1+\sqrt{3}u)} \times \frac{1+u^2}{u} = \frac{3\sqrt{3}(1+u^2)}{4(1+\sqrt{3}u)}$$

(5)  $S'(u) = \frac{3\sqrt{3}(u+\sqrt{3})(\sqrt{3}u-1)}{4(1+\sqrt{3}u)^2}$

$0 < \theta < \frac{2\pi}{3}$  のとき  $0 < u < \sqrt{3}$  だから  $S(u)$  の増減を図るようにならう

|        |   |     |                      |     |            |
|--------|---|-----|----------------------|-----|------------|
| $u$    | 0 | ... | $\frac{1}{\sqrt{3}}$ | ... | $\sqrt{3}$ |
| $S(u)$ | - | 0   | +                    |     |            |
| $S(u)$ | ↘ |     | ↗                    |     |            |

$$S\left(\frac{1}{\sqrt{3}}\right) = \frac{3\sqrt{3}\left(1 + \frac{1}{3}\right)}{4(1+1)} = \frac{\sqrt{3}}{2}$$

$u = \frac{1}{\sqrt{3}}$ ,  $\theta = \frac{\pi}{3}$  のとき  $S(u)$  は  
 $\frac{\pi}{2}$  小となり  $\frac{\pi}{2}$  小値  $\frac{\sqrt{3}}{2}$