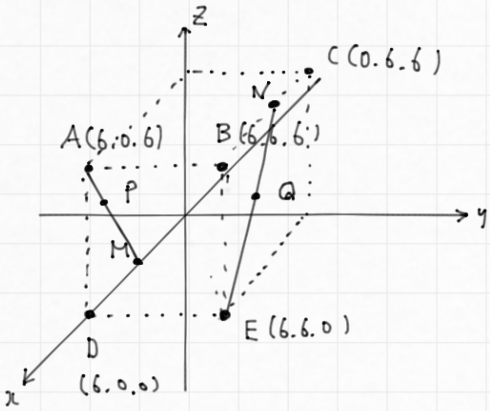
近畿大2021後①



(1) $\overrightarrow{OM} = \frac{1}{2}\overrightarrow{OD} = \begin{pmatrix}3\\0\\0\end{pmatrix}$

$\overrightarrow{ON} = \frac{1}{3}\overrightarrow{OB} + \frac{2}{3}\overrightarrow{OC} = \begin{pmatrix}2\\2\\2\end{pmatrix} + \begin{pmatrix}0\\4\\4\end{pmatrix} = \begin{pmatrix}2\\6\\6\end{pmatrix}$

$\overrightarrow{OP} = (1-s)\overrightarrow{OA} + s\overrightarrow{OM} = (1-s)\begin{pmatrix}6\\0\\6\end{pmatrix} + s\begin{pmatrix}3\\0\\0\end{pmatrix}$

$= (\mathbf{-3s+6},\ \mathbf{0},\ \mathbf{-6s+6})$

$\overrightarrow{OQ} = (1-t)\overrightarrow{ON} + t\overrightarrow{OE} = (1-t)\begin{pmatrix}2\\6\\6\end{pmatrix} + t\begin{pmatrix}6\\6\\0\end{pmatrix} = \begin{pmatrix}2+4t\\6\\6-6t\end{pmatrix}$

(2) $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = (\mathbf{3s+4t-4},\ \mathbf{6},\ \mathbf{6s-6t})$

(3) $\overrightarrow{AM} = \overrightarrow{OM} - \overrightarrow{OA} = \begin{pmatrix}3\\0\\0\end{pmatrix} - \begin{pmatrix}6\\0\\6\end{pmatrix} = \begin{pmatrix}-3\\0\\-6\end{pmatrix}$

$\overrightarrow{AM}\cdot\overrightarrow{PQ} = 0$ に成分を代入

$\begin{pmatrix}-3\\0\\-6\end{pmatrix}\cdot\begin{pmatrix}4t+3s-4\\6\\6s-6t\end{pmatrix} = -12t-9s+12-36s+36t = -45s+24t+12 = 0$

$s = \mathbf{\frac{8}{15}}t + \mathbf{\frac{4}{15}}$

(4) QからAMに下ろした垂線の足がPとなるとき (3)の条件が成り立つ

このとき

$|\overrightarrow{PQ}|^2 = (4t+3s-4)^2 + 6^2 + (6s-6t)^2 = 16t^2+9s^2+16+24st-32t-24s+36+36s^2-72st+36t^2$

$= 52t^2 - 48st + 45s^2 - 32t - 24s + 52$

$= 52t^2 - \overset{16}{\cancel{48}}t\left(\frac{8}{\underset{5}{\cancel{15}}}t + \frac{4}{\underset{5}{\cancel{15}}}\right) + \overset{3}{\cancel{45}}\frac{(8t+4)^2}{\underset{5}{\cancel{15^2}}} - 32t - \overset{8}{\cancel{24}}\left(\frac{8}{\underset{5}{\cancel{15}}}t + \frac{4}{\underset{5}{\cancel{15}}}\right) + 52$

$= 52t^2 - \frac{128}{5}t^2 - \cancel{\frac{64}{5}t} + \frac{64}{5}t^2 + \cancel{\frac{64}{5}t} + \frac{16}{5} - 32t - \frac{64}{5}t - \frac{32}{5} + 52$

$= \frac{196}{5}t^2 - \frac{224}{5}t + \frac{244}{5}$

$= \frac{28}{5}(7t^2 - 8t) + \frac{244}{5} = \frac{28\times7}{5}\left(t - \frac{4}{7}\right)^2 + 36$

$t = \frac{4}{7}$ のとき $|\overrightarrow{PQ}|$ は最小値 6

このとき △AMQの面積は $\frac{1}{2}|\overrightarrow{AM}|\times 6 = \frac{1}{2}\times\sqrt{(-3)^2+6^2}\times 6 = \mathbf{9\sqrt{5}}$

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$\Sigma x = 15 + a + b = 2 \times 10 \qquad a + b = 5$

$\Sigma y = 25 + a + c = 3 \times 10 \qquad a + c = 5$

$\Sigma z = 15 + a + b + c$

(1) $a = 3$ のとき　$b = 2$.　$c = 2$

$\Sigma x^2 = 1 + 4 + 4 + 9 + 9 + 1 + 4 + 4 + 9 + 1 = 46$

$S_x^2 = \overline{x^2} - \bar{x}^2 = 4.6 - 4 = 0.6$

	A	B	C	D	E	F	G	H	I	J
x	1	2	2	3	3	1	2	2	3	1
$x - \bar{x}$	-1	0	0	1	1	-1	0	0	1	-1
y	4	3	4	2	2	4	3	4	1	3
$y - \bar{y}$	1	0	1	-1	-1	1	0	1	-2	0
$(x-\bar{x})(y-\bar{y})$	-1	0	0	-1	-1	-1	0	0	-2	0

$-6 \qquad S_{xy} = -0.6$

	A	B	C	D	E	F	G	H	I	J	合計	平均
x	1	2	2	a	3	1	$5-a$	2	3	1	20	2
x^2	1	4	4	a^2	9	1	$(5-a)^2$	4	9	1	$2a^2-10a+58$	
y	4	3	4	2	$5-a$	4	3	4	1	a	30	3
y^2	16	9	16	4	$(5-a)^2$	16	9	16	1	a^2	$2a^2-10a+112$	
$x-\bar{x}$	-1	0	0	$a-2$	1	-1	$3-a$	0	1	-1		
$y-\bar{y}$	1	0	1	-1	$2-a$	1	0	1	-2	$a-3$		
$(x-\bar{x})(y-\bar{y})$	-1	0	0	$2-a$	$2-a$	-1	0	0	-2	$3-a$	$3-3a$	

$S_y^2 - S_x^2 = (\overline{y^2} - \bar{y}^2) - (\overline{x^2} - \bar{x}^2) = \dfrac{\cancel{2a^2} - \cancel{10a} + 112}{10} - \dfrac{\cancel{2a^2} - \cancel{10a} + 58}{10} - 9 + 4 = 5.4 - 5 = 0.4$

$S_{xy} = \dfrac{3 - 3a}{10} = \dfrac{-3}{10}a + \dfrac{3}{10}$

(3) $a = 4$ のとき　$b = c = 1$

	A	B	C	D	E	F	G	H	I	J	
z	3	4	4	2	1	1	1	1	d	4	$21 + d$
z^2	9	16	16	4	1	1	1	1	d^2	16	$65 + d^2$

	A	B	C	D	E	F	G	H	I	J	
x	1	2	2	4	3	1	1	2	3	1	
y	4	3	4	2	1	4	3	4	1	4	
z	3	4	4	2	1	1	1	1	4	4	
yz	12	12	16	4	1	4	3	4	4	16	76
xz	3	8	8	8	3	1	1	2	12	4	50

$S_z^2 = \overline{z^2} - \bar{z}^2$

$1.85^2 = \dfrac{65 + d^2}{10} - \left(\dfrac{21 + d}{10}\right)^2$

$\therefore d = 4$

$\bar{z} = \dfrac{21 + 4}{10} = 2.5$

$S_{yz} = \overline{yz} - \bar{y} \cdot \bar{z} = 7.6 - 3 \times 2.5 = 0.1$

$S_{zx} = \overline{xz} - \bar{x}\,\bar{z} = 5 - 2 \times 2.5 = 0$

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(1) $f(2) = 4 - \frac{8}{3}a - \cancel{16} + 12a + \cancel{16} - 16a = -\frac{20}{3}a + 4$

$f(a) = \frac{1}{4}a^4 - \frac{1}{3}a^4 - 2a^3 + 3a^3 + 4a^2 - 8a^2 = -\frac{1}{12}a^4 + a^3 - 4a^2$

$f'(x) = x^3 - (a+6)x^2 + 2(3a+4)x - 8a$

$f'(2) = 8 - \cancel{4a} - 24 + \cancel{12a} + 16 - \cancel{8a} = 0$

(2) $f'(1) = 1 - a - 6 + 6a + 8 - 8a = -3a + 3$ 、 $f(1) = \frac{1}{4} - \frac{a}{3} - 2 + 3a + 4 - 8a = -\frac{16}{3}a + \frac{9}{4}$

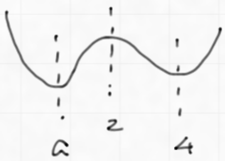
$y = (-3a+3)(x-1) - \frac{16}{3}a + \frac{9}{4}$

$= (-3a+3)x - \frac{7}{3}a - \frac{3}{4}$

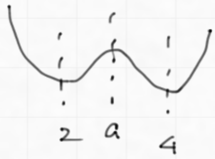
(3) $f'(x) = (x-2)(x^2 - (a+4)x + 4a) = (x-2)(x-4)(x-a)$

$x \neq 2, 4$ のとき 極大値は存在するので 極大値が存在しないのは $a = 2, 4$ の 2個 その和は 6

(4) $a < 2$ のとき $f(x)$ は $x = 2$ で極大



$2 < a < 4$ のとき、$f(x)$ は $x = a$ で極大



(5) $a = 3$ のとき、$(2, f(2))$ における接線は $f'(2) = 0$ だから $y = f(2) = -16$

このとき $f(x) = -16$ を解く

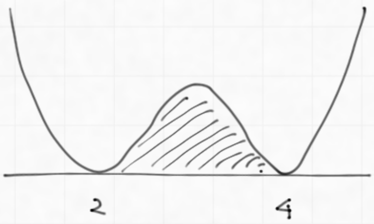
$\frac{1}{4}x^4 - 3x^3 + 13x^2 - 24x = -16$

$x^4 - 12x^3 + 52x^2 - 96x + 64 = 0$

$(x-2)(x^3 - 10x^2 + 32x - 32) = 0$

$(x-2)(x-4)(x^2 - 6x + 8) = 0$

$(x-2)^2(x-4)^2 = 0$



面積 $= \int_2^4 \frac{1}{4}x^4 - 3x^3 + 13x^2 - 24x + 16 \, dx$

$= \frac{1}{4}\int_2^4 (x-2)^2(x-4)^2 \, dx$

($x - 2 = t$ とすると $\frac{dt}{dx} = 1$,

x	$2 \to 4$
t	$0 \to 2$

)

$= \frac{1}{4}\int_0^2 t^2(t-2)^2 dt = \frac{1}{4}\int_0^2 t^4 - 4t^3 + 4t^2 \, dt$

$= \frac{1}{4}\left[\frac{1}{5}t^5 - t^4 + \frac{4}{3}t^3\right]_0^2 = \frac{2^{\cancel{5}\,3}}{\cancel{20}\,5} - \frac{2^{\cancel{4}\,2}}{\cancel{4}} + \frac{2^3}{3} = \frac{4}{15}(6 - 15 + 10) = \frac{4}{15}$