琉球大学2022

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(1) $1+\tan^2\alpha = \dfrac{1}{\cos^2\alpha}$ より $\cos\alpha = \pm\dfrac{1}{\sqrt{26}}$　　$\sin\alpha = \cos\alpha\tan\alpha = \pm\dfrac{5}{\sqrt{26}}$

$\sin 2\alpha = 2\cos\alpha\sin\alpha = 2\times\left(\pm\dfrac{1}{\sqrt{26}}\right)\times\left(\pm\dfrac{5}{\sqrt{26}}\right) = \dfrac{5}{13}$

(2) $2^{2021} \leqq n < 2^{2022}$

$2021\log_{10}2 \leqq \log_{10}n < 2022\log_{10}2$

$608.321 \leqq \log_{10}n < 608.622$

$10^{608.321} \leqq n < 10^{608.622}$　　**609桁の数**

(3)

$$
\begin{array}{r|rrrrr}
 & & 1 & -1 & a-b+1 & \\
\hline
1\ \ 1\ \ b & 1 & 0 & a & -2 & 3 \\
 & 1 & 1 & b & & \\
\hline
 & & -1 & a-b & -2 & \\
 & & -1 & -1 & -b & \\
\hline
 & & & a-b+1 & b-2 & 3 \\
 & & & a-b+1 & a-b+1 & ab-b^2+b \\
\hline
 & & & & 2b-a-3 & 3-ab+b^2-b
\end{array}
$$

$2b-a-3=0$ かつ

$3-ab+b^2-b=0$

連立して

$3-(2b-3)b+b^2-b=0$

$b^2-2b-3=0$　　$b=3,-1$

$a=3,-5$

$(a,b)=$ **(3, 3), (−5, −1)**

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(1) C_1上の点 $(t, -t^2+2t)$ における接線は

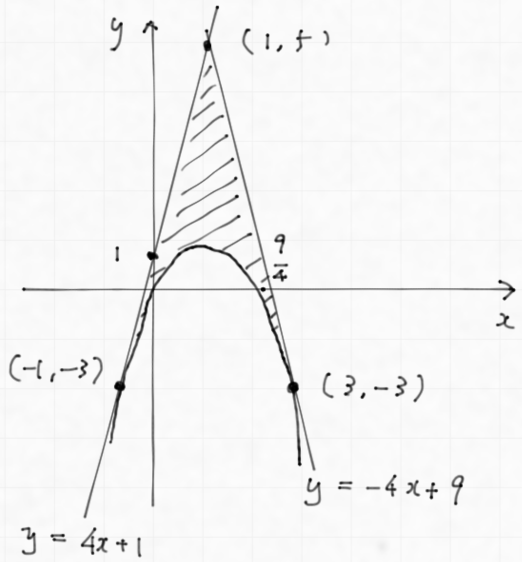
$y' = -2x+2$ だから　　　$y = (-2t+2)(x-t) - t^2+2t$

つまり. $y = 2(1-t)x + t^2$

これが C_2 と接するので連立し. 判別式Dをとる.

$$2x^2-4x+9 = 2(1-t)x+t^2$$
$$2x^2+2(t-3)x-t^2+9=0$$
$$D/4 = (t-3)^2+2(t^2-9)=0$$
$$3t^2-6t-9=0$$
$$t^2-2t-3=0$$
$$t=-1, 3$$

接線は　$y = 4x+1$,　$y = -4x+9$



(2) (1)の2つの接線の交点は $(1, 5)$.

面積をもとめるのは右図の斜線部

面積をSとして

$$S = \int_{-1}^{1} 4x+1-(-x^2+2x)\,dx + \int_{1}^{3} -4x+9-(-x^2+2x)\,dx$$
$$= \int_{-1}^{1}(x+1)^2dx + \int_{1}^{3}(x-3)^2dx$$
$$= \left[\frac{1}{3}(x+1)^3\right]_{-1}^{1} + \left[\frac{1}{3}(x-3)^3\right]_{1}^{3}$$
$$= \frac{8}{3} + \frac{8}{3} = \frac{16}{3}$$