摂南大2022

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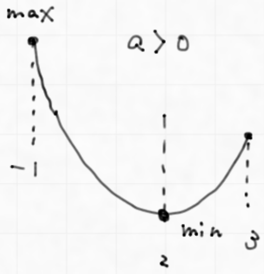
(1) $x+\frac{1}{x}=\frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}}+\frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}}=\frac{7-2\sqrt{21}+3+7+2\sqrt{21}+3}{7-3}=\frac{20}{4}=\mathbf{5}$

$x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2=5^2-2=\mathbf{23}$

$x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right)=125-15=\mathbf{110}$



(2) $y=a(x-2)^2-4a+b$

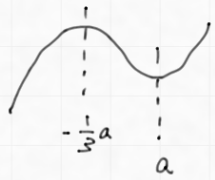
$a>0$ のとき　$x=2$ で最小　$x=-1$ で最大

$-4a+b=-2$,　$5a+b=4$　　$(a,b)=\left(\frac{2}{3},\frac{2}{3}\right)$

$a<0$ のとき　$x=2$ で最大　$x=-1$ で最小

$-4a+b=4$,　$5a+b=-2$　　$(a,b)=\left(-\frac{2}{3},\frac{4}{3}\right)$

(3) $f(x)=3x^2-2ax-a^2=(3x+a)(x-a)$



$\alpha=-\frac{1}{3}a$, $\beta=a$　　$\alpha\beta=-\frac{a^2}{3}$　$\alpha+\beta=\frac{2a}{3}$

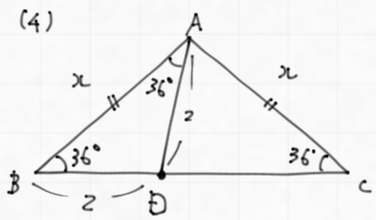
$\alpha-\beta=-\frac{4}{3}a$

$f(\alpha)-f(\beta)=\alpha^3-a\alpha^2-a^2\alpha-\beta^3+a\beta^2+a^2\beta$

$=(\alpha-\beta)\left(\alpha^2+\alpha\beta+\beta^2-a(\alpha+\beta)-a^2\right)=-\frac{4}{3}a\left(\frac{4a^2}{9}+\frac{a^2}{3}-\frac{2a^2}{3}-a^2\right)$

$=\frac{32}{27}a^3=\sqrt{2}$　　$a^3=\frac{27}{32}\sqrt{2}=3^3\cdot2^{-\frac{9}{2}}$　　$a=3^1\cdot2^{-\frac{3}{2}}=\frac{3}{2\sqrt{2}}=\frac{3\sqrt{2}}{4}$

(4)



$\angle BAC=180°-36°\times2=108°$

$\angle DAC=108°-36°=72°=\angle ADC$ だから　$\triangle CAD$ は $CA=CD$ の二等辺三角形で　$DC=AC=x$

$\triangle ABC\sim\triangle DAB$ より　$x:2+x=2:x$

$x^2-2x-4=0$　　$x=1+\sqrt{5}$

$\cos\angle BAC=\frac{x^2+x^2-(x+2)^2}{2x^2}=\frac{x^2-4x-4}{2x^2}=\frac{2x+\cancel{4}-4x-\cancel{4}}{2(2x+4)}$

$=\frac{-x}{2x+4}=\frac{-1-\sqrt{5}}{6+2\sqrt{5}}=\frac{4-4\sqrt{5}}{16}=\frac{1-\sqrt{5}}{4}$

(5) $\vec{AB} = (2, -1, 0)$ $\vec{AC} = (5, -4, 3)$, $\vec{AD} = (t+2, -4, t-5)$

$\vec{AD} = \alpha\vec{AB} + \beta\vec{AC}$ に 数値を代入

$$\begin{pmatrix} t+2 \\ -4 \\ t-5 \end{pmatrix} = \alpha\begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} + \beta\begin{pmatrix} 5 \\ -4 \\ 3 \end{pmatrix}$$

$$\begin{cases} t+2 = 2\alpha + 5\beta \\ -4 = -\alpha - 4\beta \\ t-5 = 3\beta \end{cases}$$

連立して $\alpha = \frac{10}{3}$, $\beta = \frac{1}{6}$, $t = \frac{11}{2}$

$\vec{AD} = \frac{10}{3}\vec{AB} + \frac{1}{6}\vec{AC}$

(6) ${}_3H_{12} = {}_{14}C_2 = \frac{14 \cdot 13}{2} = 91$ 通り

全て異なり $x < y < z$ となるのは

$(x, y, z) = (0, 1, 11), (0, 2, 10), (0, 3, 9), (0, 4, 8), (0, 5, 7)$
$(1, 2, 9), (1, 3, 8), (1, 4, 7), (1, 5, 6)$
$, (2, 3, 7), (2, 4, 6), (3, 4, 5)$ の12通り

よって全て異なるのは $12 \times 3! = 72$ 通り

91通り $\begin{cases} 3つ同じ (4, 4, 4) の 1通り \\ 2つ同じ \quad 91 - 72 - 1 = 18 通り \\ 全て異なる \quad 72 通り \end{cases}$

左のうち $x \leqq y \leqq z$ を満たすもの

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$18 \div 3 = 6$

$72 \div 3! = 12$

合計 19 通り

(7) $\tan\frac{\theta}{2} = t$ とおく

$1 + \tan^2\frac{\theta}{2} = \frac{1}{\cos^2\frac{\theta}{2}}$ より $\cos^2\frac{\theta}{2} = \frac{1}{1+t^2}$

$\cos\theta = 2\cos^2\frac{\theta}{2} - 1 = \frac{2}{1+t^2} - 1 = \frac{1-t^2}{1+t^2}$

$\tan\theta = \frac{2t}{1-t^2}$ $\sin\theta = \tan\theta\cos\theta = \frac{2t}{1+t^2}$

$5\cos\theta - \sqrt{3}\sin\theta + 1 > 0$ に 上の値を代入

$$\frac{5(1-t^2)}{1+t^2} - \frac{2\sqrt{3}t}{1+t^2} + 1 > 0$$

$5 - 5t^2 - 2\sqrt{3}t + 1 + t^2 > 0$

$2t^2 + \sqrt{3}t - 3 < 0$

$(2t - \sqrt{3})(t + \sqrt{3}) < 0$

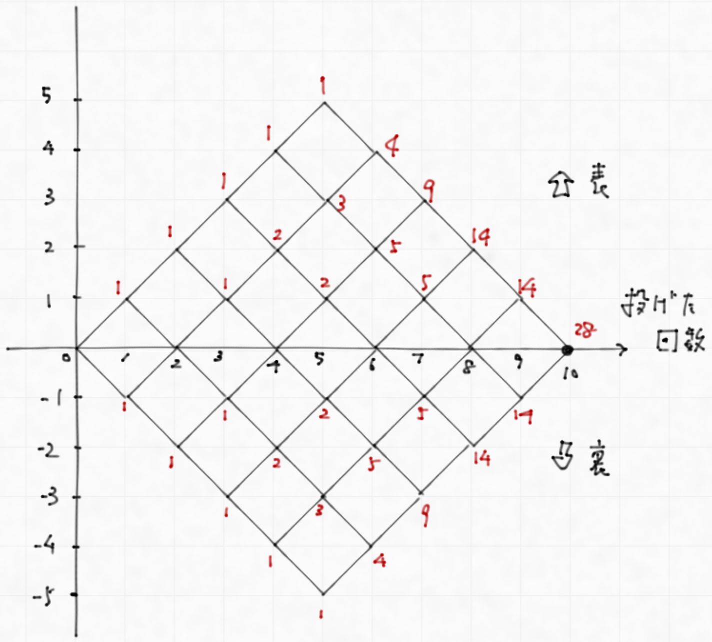
$-\sqrt{3} < t < \frac{\sqrt{3}}{2}$

2

(1) $\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^5 \times {}_{10}C_5 = \frac{\cancel{10}^{\,2}\cdot 9\cdot \cancel{8}\cdot 7\cdot \cancel{6}^{\,2}}{\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 2^{\cancel{10}\,8}} = \frac{63}{256}$

(2) $\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 \times {}_4C_2 \times \left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^3 \times {}_6C_3 = \frac{\cancel{4}\cdot 3\cdot \cancel{6}\cdot 5\cdot \cancel{4}^{\,2}}{\cancel{2}\cdot\cancel{3}\cdot\cancel{2}\cdot 2^{\cancel{10}\,7}} = \frac{15}{128}$

(3)



左図より

$$\frac{28}{2^{10}} = \frac{7}{256}$$