千葉工業大2020 A日程

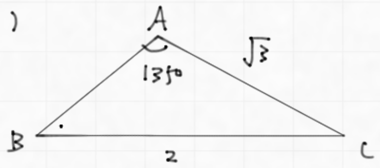
1

(1) $Z = \dfrac{23-6i}{1-2i} = \dfrac{23-6i}{1-2i} \times \dfrac{1+2i}{1+2i} = \mathbf{7+8i}$

(2) $3! \times 3! \times 2 = \mathbf{72}$ 通り

(3) $\dfrac{{}_1C_1 \times {}_4C_1 \times {}_5C_1}{{}_{10}C_3} = \mathbf{\dfrac{1}{6}}$

(4)



正弦定理より $\dfrac{2}{\sin 135°} = \dfrac{\sqrt{3}}{\sin B}$ $\quad \sin B = \mathbf{\dfrac{\sqrt{6}}{4}}$

(5) $\left(\dfrac{1}{4}\right)^x = X$ として $\dfrac{1}{2}X^2 \times \left(\dfrac{1}{16}\right)^{-1} - 9X + 1 = 0 \iff 8X^2 - 9X + 1 = 0 \iff (8X-1)(X-1) = 0$

$X = \dfrac{1}{8}, 1 = \left(\dfrac{1}{4}\right)^x \qquad \therefore x = 0, \mathbf{\dfrac{3}{2}}$

(6) 公差を d として

$$\frac{-10 + (-10+2d)}{\cancel{2}} \times 3 = \frac{-10 + (-10+9d)}{\cancel{2}} \times 10$$

$$-60 + 6d = -200 + 90d$$

$$84d = 140 \qquad d = \frac{140}{84} = \frac{35}{21} = \mathbf{\frac{5}{3}}$$

(7) $|\vec{a} - \vec{b}|^2 = 9 - 2\vec{a}\cdot\vec{b} + |\vec{b}|^2 = 25$ $\quad$ (−

$|\vec{a} + \vec{b}|^2 = 9 + 2\vec{a}\cdot\vec{b} + |\vec{b}|^2 = 49$ $\quad$ (+

$4\vec{a}\cdot\vec{b} = 24$ $\qquad \vec{a}\cdot\vec{b} = \mathbf{6} \qquad |\vec{b}| = \mathbf{2\sqrt{7}}$

(8) $\displaystyle\int_{-1}^{0} x^2(-2x+x)\,dx + \int_{0}^{2} x^2(2x+x)\,dx = \left[-\frac{1}{4}x^4\right]_{-1}^{0} + \left[\frac{3}{4}x^4\right]_{0}^{2}$

$= +\dfrac{1}{4} + 12 = \mathbf{\dfrac{49}{4}}$

2

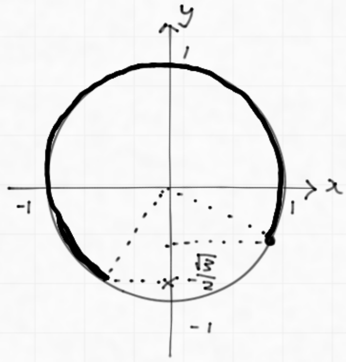
(1) $F = 12\sqrt{2}\sin 3x + 4\sqrt{6}\cos\left(3x + \frac{2}{3}\pi\right)$

$= 12\sqrt{2}\sin 3x + 4\sqrt{6}\left(\cos 3x \cos\frac{2}{3}\pi - \sin 3x \sin\frac{2}{3}\pi\right)$

$= 12\sqrt{2}\sin 3x + 4\sqrt{6}\left(-\frac{1}{2}\cos 3x - \frac{\sqrt{3}}{2}\sin 3x\right)$

$= \mathbf{6\sqrt{2}}\sin 3x - \mathbf{2\sqrt{6}}\cos 3x$

$= \mathbf{4\sqrt{6}}\sin\left(3x - \frac{\mathbf{1}}{\mathbf{6}}\pi\right)$



$0 \leqq x \leqq \frac{\pi}{2}$ のとき

$-\frac{1}{6}\pi \leqq 3x - \frac{1}{6}\pi \leqq \frac{4}{3}\pi$ だから

$-\frac{\sqrt{3}}{2} \leqq \sin\left(3x - \frac{1}{6}\pi\right) \leqq 1$

$\mathbf{-6\sqrt{2}} \leqq 4\sqrt{6}\sin\left(3x - \frac{1}{6}\pi\right) \leqq 4\sqrt{6}$

(2) $G = (1 + \log_2 x)(2\log_2 x - 1)(3 - \log_2 x)$

$G = 0$ となるのは $\log_2 x = -1, \frac{1}{2}, 3$ 　すなわち $x = 2^{-1}, 2^{\frac{1}{2}}, 2^{3} = \frac{1}{2}, \sqrt{2}, 8$

最も小さいのは $x = \mathbf{\frac{1}{2}}$

$G = (1+t)(2t-1)(3-t) = \mathbf{-2}t^3 + \mathbf{5}t^2 + \mathbf{4}t \mathbf{-3}$

$x \geqq \frac{1}{2}$ のとき　$t \geqq -1$

$G' = -6t^2 + 10t + 4 = -2(3t+1)(t-2)$

$G' = 0$ となるのは $t = -\frac{1}{3}, 2$

t	-1	$\cdots$	$-\frac{1}{3}$	$\cdots$	2	$\cdots$
G'		$-$	0	$+$	0	$-$
G		↘		↗		↘

$t = -1$ のとき　$G = 0$

$t = 2$ のとき　$G = 3 \times 3 \times 1 = 9$ 　　$t = 2$, すなわち $x = \mathbf{4}$ のとき　G は最大値 $\mathbf{9}$

$(\because \log_2 x = 2)$

3 (1) $P(x) = (x^2-4)Q(x) + 8x+3$　　∴ $P(-2) = 0-16+3 = -13$

$= (x+2)(x-3)R(x) + 6x + k$　　$= -12 + k$

$k = -1$

$= (x^2-4)(x-3)S(x) + ax^2+bx+c$

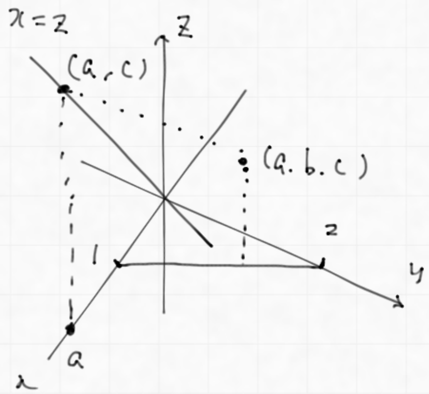
$P(-2) = -13 = 4a-2b+c$

$P(2) = 8\times2+3 = 4a+2b+c$

$P(3) = 6\times3-1 = 9a+3b+c$

これらを連立して $a=-2,\ b=8.\ c=11$　　余りは $-2x^2+8x+11$

(2)



xy平面上で $(1.0.0), (0,2,0)$ を通る直線は

$y = -2x+2$, $z=0$.

円の中心はこの線上にあるので　$b = -2a+2$

xz平面との交線の円は $x=z$ 上にあるので　$a=c$

$R = \sqrt{(a-a)^2+(b-0)^2+(c-0)^2}$

$= \sqrt{(-2a+2)^2+a^2} = \sqrt{5a^2-8a+4}$

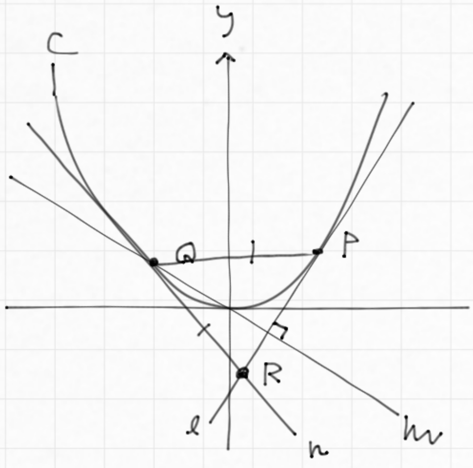
$= \sqrt{5(a-\frac{4}{5})^2+\frac{4}{5}}$

Rの最小値は $\sqrt{\frac{4}{5}} = \frac{2\sqrt{5}}{5}$, $a=\frac{4}{5}=c$

4

(1) $y' = \frac{1}{2}x$

l: $y = \frac{1}{2}p(x-p) + \frac{1}{4}p^2 \iff y = \frac{1}{2}px - \frac{1}{4}p^2$



m: $y = -\frac{2}{p}x$ とlの交点は

$-\frac{2}{p}x = \frac{1}{2}px - \frac{1}{4}p^2$ より $y = -\frac{p^2}{p^2+4}$

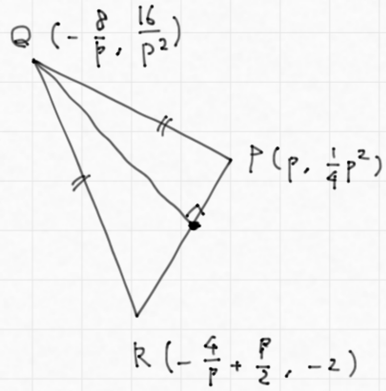
(2) $\frac{1}{4}x^2 = -\frac{2}{p}x$ より $x = \frac{-8}{p}$

n: $y = -\frac{1}{2}\times\frac{8}{p}\left(x+\frac{8}{p}\right) + \frac{16}{p^2}$

$y = -\frac{4}{p}x - \frac{16}{p^2}$

nとlの交点は $\frac{1}{2}px - \frac{1}{4}p^2 = -\frac{4}{p}x - \frac{16}{p^2}$ を解いて $x = -\frac{4}{p} + \frac{p}{2}$, $y = -2$

(3)



PQ $=$ QR のとき lとmの交点はPRの中点.

$$-\frac{p^2}{p^2+4} = \frac{\frac{1}{4}p^2 + (-2)}{2}$$

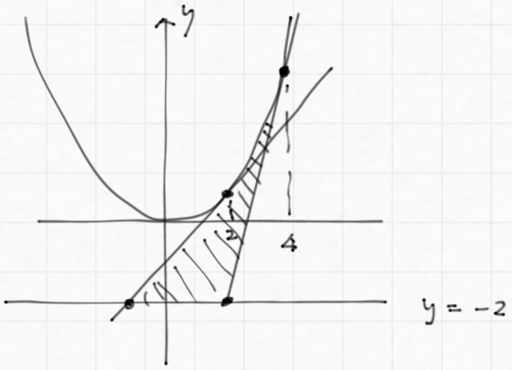
$$p^4 + 4p^2 - 32 = 0$$

$$(p^2+8)(p^2-4) = 0$$

$\therefore p = 2$

(4) $p=2$ のとき R$(-1, -2)$ だから $-\frac{4}{p} + \frac{p}{2} = 1$ より $p^2 - 2p - 8 = 0$ $p = -2, 4$

$\therefore p = 4$



PRの通過領域は左図のようになり

$$\frac{1}{2}\cdot 3\times 3 + \int_2^4\left(\frac{1}{4}x^2+2\right)dx - \frac{1}{2}\cdot 3\cdot 6 = \frac{25}{6}$$