日本大工 CA 2023

1 (1) $(2^3)^2 \times 2^{-2} + \log_2 \frac{12}{5} + \log_2 \frac{80}{3}$

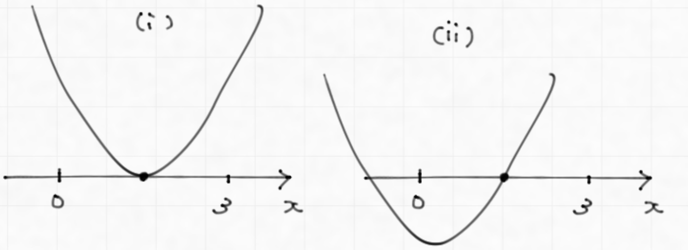
$= 2^6 \times 2^{-2} + \log_2 \frac{\cancel{12}^{4}}{\cancel{5}} \times \frac{\cancel{80}^{16}}{\cancel{3}} = 2^4 + \log_2 2^6 = 16 + 6 = \mathbf{22}$

(2) $2\cos^2\theta - \sin\theta - 1 = 0 \iff 2(1-\sin^2\theta) - \sin\theta - 1 = 0$

$\iff 2\sin^2\theta + \sin\theta - 1 = 0 \iff (2\sin\theta - 1)(\sin\theta + 1) = 0 \iff \sin\theta = \frac{1}{2}, -1$

$\iff \theta = \frac{\pi}{6}, \frac{5}{6}\pi, \frac{3}{2}\pi$

$\left(\frac{\pi}{6} + \frac{5}{6}\pi + \frac{3}{2}\pi\right) \times \frac{1}{3} = \mathbf{\frac{5}{6}\pi}$



(3) $f(x) = x^2 - 2ax + a^2 - a + 1$ とおく

$f(x) = 0$ が $0 < x < 3$ の範囲で1つの解をもつ条件は

(i) $0 < x < 3$ で重解をもつ　(ii) $0 < x < 3$ に1つ他に1つの解をもつ

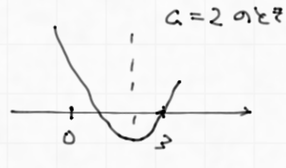
(i) 軸 $x = a$ が $0 < x < 3$ の範囲内 かつ 判別式 $D = 0$

$0 < a < 3$,　$\frac{D}{4} = a^2 - a^2 + a - 1 = 0$　　$a = 1$

(ii) $f(0)$ と $f(3)$ が異符号

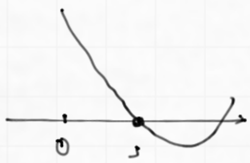
$f(0) = a^2 - a + 1 = (a - \frac{1}{2})^2 + \frac{3}{4} > 0$

$f(3) = 9 - 6a + a^2 - a + 1 = (a-5)(a-2) < 0 \iff 2 < a < 5$



$f(3) = 0$ のとき　$a = 2$ または $a = 5$

$a = 2$ のときは $0 < x < 3$ で1つの解をもつが、$a = 5$ のときはもたない (右図)



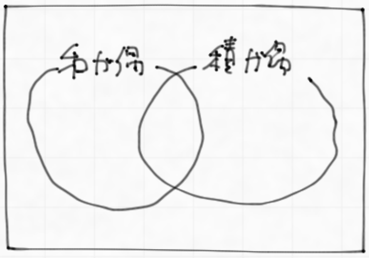
$\therefore$ $\mathbf{a = 1}$ または $\mathbf{2 \leqq a < 5}$

3

(1) 和が偶数 $\frac{3}{6}\times\frac{3}{6}\times 2 = \frac{1}{2}$

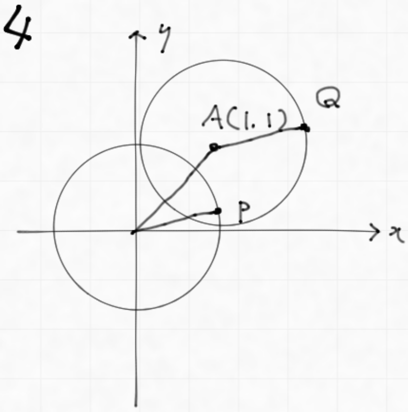
(2) 積が偶数 $1-\frac{3}{6}\times\frac{3}{6} = \frac{3}{4}$

(3) 積が偶数かつ和が偶数 $\frac{3}{6}\times\frac{3}{6} = \frac{1}{4}$



積が偶数のとき和が偶数 $= \dfrac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$

4



(1) $\vec{OQ} = \vec{OA} + \vec{OP} = \begin{pmatrix}1\\1\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\2\end{pmatrix}$

$\vec{OR} = \vec{OP} + \vec{OQ} = \begin{pmatrix}0\\1\end{pmatrix} + \begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}1\\3\end{pmatrix}$

(2) 平行移動 $(x-1)^2+(y-1)^2=1$

(3) $\vec{OR} = \vec{OP} + \vec{OA} + \vec{OP} = \vec{OA} + 2\vec{OP}$

平行移動と拡大 $(x-1)^2+(y-1)^2=4$

5

(1) $\vec{CH}\cdot\vec{OA} = \begin{pmatrix}a+1\\b-1\\c-3\end{pmatrix}\cdot\begin{pmatrix}2\\0\\0\end{pmatrix} = 2a+2 = 0$ $a=-1$

(2) $\vec{CH}\cdot\vec{OB} = \begin{pmatrix}a+1\\b-1\\c-3\end{pmatrix}\cdot\begin{pmatrix}1\\4\\1\end{pmatrix} = a+1+4b-4+c-3=0$ $a=-1$ を代入整理して $4b+c=7$

$\vec{OH} = \alpha\vec{OA} + \beta\vec{OB} = \begin{pmatrix}2\alpha\\0\\0\end{pmatrix} + \begin{pmatrix}\beta\\4\beta\\\beta\end{pmatrix} = \begin{pmatrix}a\\b\\c\end{pmatrix}$

$2\alpha+\beta=a$, $4\beta=b$, $\beta=c$

$a=-1$, $c=7-4b$ を代入 $2\alpha+\beta=-1$, $4\beta=b$, $\beta=7-4b$

$4(7-4b)=b \Leftrightarrow 17b=28$ $b=\frac{28}{17}$, $\beta=\frac{7}{17}$ $\alpha=-\frac{12}{17}$, $c=\frac{7}{17}$

$\vec{CH} = \begin{pmatrix}a+1\\b-1\\c-3\end{pmatrix} = \begin{pmatrix}-1+1\\\frac{28}{17}-1\\\frac{7}{17}-3\end{pmatrix} = \begin{pmatrix}0\\\frac{11}{17}\\-\frac{44}{17}\end{pmatrix} = \left(0,\ \frac{11}{17},\ -\frac{44}{17}\right)$

(3) $\triangle OAB$

$|\vec{OA}|=2$, $|\vec{OB}| = \sqrt{1+16+1} = 3\sqrt{2}$

$\vec{OA}\cdot\vec{OB} = 2$ $\cos\angle AOB = \frac{2}{2\cdot 3\sqrt{2}} = \frac{\sqrt{2}}{6}$ $\sin\angle AOB = \sqrt{1-\left(\frac{\sqrt{2}}{6}\right)^2} = \frac{\sqrt{17}}{3\sqrt{2}}$

$\triangle OAB = \cancel{\frac{1}{2}}\times\cancel{2}\times\cancel{3\sqrt{2}}\times\frac{\sqrt{17}}{\cancel{3\sqrt{2}}} = \sqrt{17}$

$|\vec{CH}| = \sqrt{\left(\frac{11}{17}\right)^2+\left(\frac{44}{17}\right)^2} = \frac{11}{17}\sqrt{17}$ $V = \frac{1}{3}\times\sqrt{17}\times\frac{11}{17}\sqrt{17} = \frac{11}{3}$

6 $f(x) = x^2 + 2x + a + \int_1^x (t^2 + 2b)\,dt$

(1) $f(2) = 4 + 4 + a + \left[\frac{1}{3}t^3 + 2bt\right]_1^2 = 8 + a + \frac{8}{3} + 4b - \frac{1}{3} - 2b = a + 2b + \frac{31}{3}$

(2) $f'(x) = 2x + 2 + x^2 + 2b = x^2 + 2x + 2b + 2$

(3) $f'(2) = 4 + 4 + 2b + 2 = 0$ $\quad b = -5$

$f(2) = a + 2b + \frac{31}{3} = \frac{10}{3}$ $\quad a = 10 - \frac{21}{3} = 3$