$\boxed{4}$ (1)

$$
\begin{array}{r|l}
2 & 234234 \\
3 & 117117 \\
3 & 39039 \\
13 & 13013 \\
7 & 1001 \\
11 & 143 \\
 & 13
\end{array}
$$

$234234 = 2^1 \times 3^2 \times 7^1 \times 11^1 \times 13^2$

$2 \times 3 \times 2 \times 2 \times 3 = \underline{72}$ 個

3より大きな素数は 7, 11, 13.

$n = 7 \times 11 \times 13 = 1001$

$$
\begin{array}{r|l}
8 & 1001 \\
8 & 125 \cdots 1 \\
8 & 15 \cdots 5 \\
 & 1 \cdots 7
\end{array}
$$

$1001 = \underline{1751_{(8)}}$

(2)

A	B	C	D	E
97	86	66	x	$x+10$

$x+10 \geq 86$, $x \leqq 86$ $\Leftrightarrow$ $\underline{76 \leqq x \leqq 86}$

平均は $\dfrac{259+2x}{5}$

分散は $V = E(X^2) - \overline{E(X)}^2 = \dfrac{1}{5}\left(97^2+86^2+66^2+x^2+(x+10)^2\right) - \left(\dfrac{259+2x}{5}\right)^2$

$= \dfrac{2}{5}x^2 + 4x + \dfrac{1}{5}(97^2+86^2+66^2+10^2) - \dfrac{4}{25}x^2 - \dfrac{1036}{25}x - \left(\dfrac{259}{5}\right)^2$

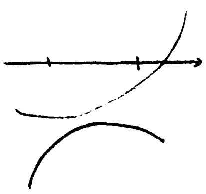
$= \dfrac{6}{25}x^2 - \dfrac{936}{25}x + \cdots$

$= \dfrac{6}{25}(x-78)^2 + \cdots$

259

$x = 78$ のとき…

(3) $f(x) = bx^2 - 2ax - b - 4$



b>0のとき. $f(-1-\sqrt{2}) \leqq 0$ かつ $f(1+\sqrt{2}) \leqq 0$

b>0 のとき

$-1-\sqrt{2} \leqq \dfrac{a}{b} \leqq 1+\sqrt{2}$ のとき $f\left(\dfrac{a}{b}\right) \leqq 0$

上記以外のとき $f(-1-\sqrt{2}) \leqq 0$ かつ $f(1-\sqrt{2}) \leqq 0$

$\boxed{5}$ (1) $f(x) = \cos 3x + \cos 2x + \cos x$

$= 2\cos 2x \cos x + \cos 2x = \cos 2x(2\cos x + 1) = 0$

$2x = \frac{\pi}{2}, \frac{3}{2}\pi, \quad x = \frac{2}{3}\pi$

$\underline{x = \frac{\pi}{4}, \frac{3}{4}\pi, \frac{2}{3}\pi.}$

$f(x) = \cos(2x + x) + 2\cos^2 x - 1 + \cos x$

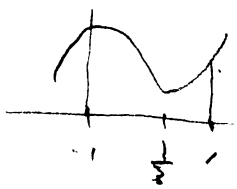
$= (2\cos^2 x - 1)\cos x - 2\sin^2 x \cos x + 2\cos^2 x - 1 + \cos x$

$= 4\cos^3 x + 2\cos^2 x - 2\cos x - 1$

$= 4X^3 + 2X^2 - 2X - 1$

$f'(x) = 12X^2 + 4X - 2 = 2(6X^2 + 2X - 1) = 2(3X - 1)(X + 1)$

$X = \frac{1}{3}, -1.$



$f(\frac{1}{3}) = 4 \cdot (\frac{1}{3})^3 + 2(\frac{1}{3})^2 - \frac{2}{3} - 1$

$= \frac{1}{27}(4 + 6 - 18 - 27) = \boxed{-\frac{35}{27}}$

(2) $\lim_{x \to +\infty} \frac{\sqrt{x}(1 + \cancel{x} - \cancel{x})}{\sqrt{1+x} + \sqrt{x}} = \lim_{x \to \infty} \frac{\sqrt{1}}{\sqrt{1 + \frac{1}{x}} + \sqrt{1}} = \underline{\frac{1}{2}}$

$f(x) = \cos^2 x.$

$\frac{\cos^2 \frac{k}{n}\pi - \cos^2 \frac{k-1}{n}\pi}{\frac{k}{n}\pi - \frac{k-1}{n}\pi} = -2\cos c \sin c$

$\cos^2 \frac{k}{n}\pi - \cos^2 \frac{k-1}{n}\pi = -\frac{\pi}{n}\sin 2c \qquad \frac{k-1}{n}\pi < c < \frac{k}{n}\pi$

$\lim \sum \frac{1}{n} \pi \sin 2 \frac{k}{n}\pi$

$4,$

$= \int_0^2 \pi \sin 2x \, dx = \frac{1}{2}[\cos x]_0^2 = -\frac{1}{2} + \frac{1}{2} = 4$

$\boxed{6}$ (1) $f(x) = g(x)$ より

$$(e^{2x}-1)^2 = 1 + \frac{2}{e^{2x}-e^{-2x}}$$

$e^{2x}=X$ として

$$(X-1)^2 = 1 + \frac{2}{X-\frac{1}{X}} = 1 + \frac{2X}{X^2-1}$$

$$X^2-2X = \frac{2X}{X^2-1}$$

$$(X-2)(X^2-1) = 2 \qquad (\because X \neq 0)$$

$$X(X^2-2X-1) = 0 \qquad X = 1+\sqrt{2} \qquad (\because X = e^{2x} > e^0 = 1)$$

$$a = \underline{\frac{1}{2}\log(1+\sqrt{2})}$$

(2) $x > 0$ のとき $e^{2x} > 1$ だから、 $e^{2x} - e^{-2x} > 0$.

したがって $a \leqq x \leqq \log k$ において $g(x)$ は常に正、したがって条件は

$$\log 6 - \frac{1}{2}\log 5 = \int_{\frac{1}{2}\log(1+\sqrt{2})}^{\log k} 1 + \frac{2}{e^{2x}-e^{-2x}}\,dx$$

$$= \int_{\frac{1}{2}\log(1+\sqrt{2})}^{\log k} 1 + \frac{2e^{2x}}{e^{4x}-1}\,dx \quad \cdots (*)$$

$\displaystyle\int \frac{e^{2x}}{e^{4x}-1}dx$ について、$e^{2x} = t$ として、 $\dfrac{dt}{dx} = 2e^{2x}$ だから

$$\int \frac{1}{t^2-1}dt = \frac{1}{2}\int \frac{1}{t-1} - \frac{1}{t+1}\,dt = \frac{1}{2}\left(\log|t-1| - \log|t+1|\right) + C$$

$$= \frac{1}{2}\log\left|\frac{t-1}{t+1}\right| + C$$

$$(*) = \left[x + \frac{1}{2}\log\left|\frac{e^{2x}-1}{e^{2x}+1}\right|\right]_{\frac{1}{2}\log(\sqrt{2}+1)}^{\log k}$$

$$= \log k - \log(\sqrt{2}+1)^{\frac{1}{2}} + \frac{1}{2}\log\left(\frac{k^2-1}{k^2+1}\right) - \frac{1}{2}\log\left(\frac{\sqrt{2}+1-1}{\sqrt{2}+1+1}\right) \qquad (\because k > 1)$$

$$= \frac{1}{2}\log k^2 - \frac{1}{2}\log(\sqrt{2}+1) + \frac{1}{2}\log\left(\frac{k^2-1}{k^2+1}\right) - \frac{1}{2}\log\frac{\sqrt{2}}{2+\sqrt{2}}$$

$$\frac{1}{2}\log 6^2 \times \frac{1}{5} = \frac{1}{2}\log k^2 \times \frac{1}{(\sqrt{2}+1)} \times \frac{k^2-1}{k^2+1} \times \frac{2+\sqrt{2}}{\sqrt{2}}$$

$$\frac{36}{5} = \frac{k^2(k^2-1)}{k^2+1} \iff (k^2-9)(5k^2+4) = 0. \qquad k^2 > 1+\sqrt{2} \text{ より}$$

$$\underline{k = 3}$$