① (1) $(1,12),(2,6),(3,4),(4,3),(6,2),(12,1)$ //

(2) $x(y+2)-y-2-12=0$

$(y+2)(x-1)=12$

$(x-1,\ y+2)=(1,12),(2,6),(3,4),(4,3),(6,2),(12,1)$

$(x,y)=(2,10),(3,4),(4,2),(5,1),\cancel{(7,0)},\cancel{(13,-1)}$ //

(3)

$4y-5x+xy=0$

$(x+4)(y-5)=-20$

$x+4\geqq 5,\quad y-5\geqq -4$

$(x+4,\ y-5)=(5,-4),(10,-2),(20,-1)$

$(x,y)=(1,1),(6,3),(16,4)$

②

$f(x)=\left[-\frac{1}{2}\cos 2t\right]_x^{2x}=-\frac{1}{2}\cos 4x+\frac{1}{2}\cos 2x$

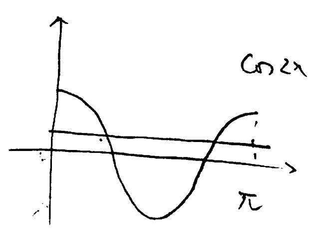
$f'(x)=2\sin 4x-\sin 2x$

$=4\sin 2x\cos 2x-\sin 2x$

$=\sin 2x(4\cos 2x-1)$

$\sin 2x=0 \qquad x=0,\ \frac{\pi}{2},\ \pi$

$\cos 2x=\frac{1}{4} \qquad x=\alpha,\ \pi-\alpha$




| $x$ | 0 | … | $\alpha$ | … | $\frac{\pi}{2}$ | … | $\pi-\alpha$ | … | $\pi$ |
|---|---|---|---|---|---|---|---|---|---|
| $f'(x)$ | 0 | + | 0 | − | 0 | + | 0 | − | 0 |
| $f(x)$ | | ↗ | | ↘ | | ↗ | | ↘ | |

$f(0)=0,\ f(\pi)=0$

$f(\frac{\pi}{2})=-\frac{1}{2}-\frac{1}{2}=-1$

$x = \frac{\pi}{2}$ のとき　最小値 $-1$

$x = \alpha$ のとき　　$\cos 2\alpha = \frac{1}{4}$.

$f(\alpha) = -\frac{1}{2}\cos 4\alpha + \frac{1}{2}\cos 2\alpha$

$= -\frac{1}{2}\left(2\cos^2 2\alpha - 1\right) + \frac{1}{2}\cos 2\alpha = -\frac{1}{2}\left(2 \times \left(\frac{1}{4}\right)^2 - 1\right) + \frac{1}{2} \times \frac{1}{4}$

$= \frac{7}{16} + \frac{1}{8} = \frac{9}{16} \quad = f(\pi - \alpha)$.

③

$b_k = a_{3k-1} = (3k-1)^2 - (3k-1) + 1 = 9k^2 - 9k + 3 = 9k(k-1) + 3$

$b_{10} = 900 - 90 + 3 = 813$

$b_{11} = 1089 - 99 + 3 = 993$

$b_{12} = 9 \times 12 \times 11 + 3 = 1191$

$\vdots$

$\ell = 4, \quad m = 11$

$b_6 = 9 \times 6 \times 5 + 3 = 273$

$b_5 = 9 \times 5 \times 4 + 3 = 183$

$b_4 = 9 \times 4 \times 3 + 3 = 111$

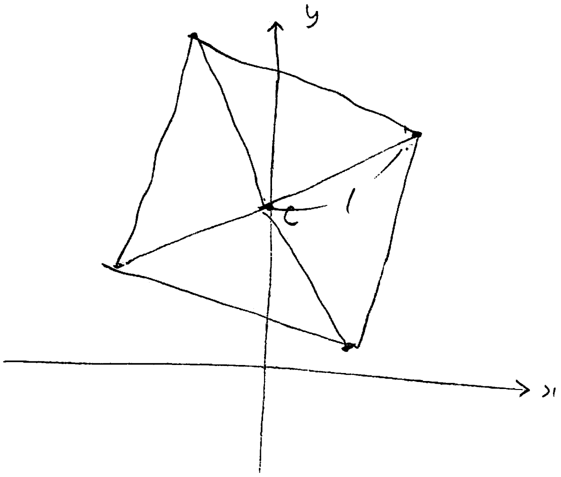
$b_3 = 9 \times 3 \times 2 + 3 = 57$

$$\sum_{k=4}^{11}(9k^2 - 9k + 3) = \frac{9}{\cancel{6}} \times 11 \times \cancel{12}^{2} \times 23 - \frac{9}{\cancel{6}} \times \cancel{3} \times \cancel{4}^{2} \times 7 + \frac{-33 - 96}{2} \times 8$$

$= 9(506 - 14) - 129 \times 4$

$= 4428 - 516 = 3912$

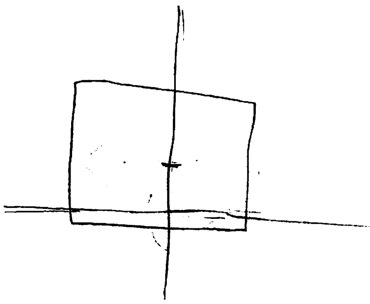
④

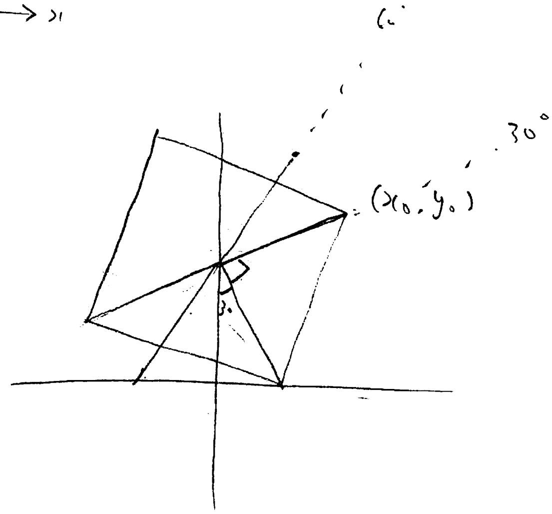



(1) $\sqrt{2}$.

(2) $-\frac{1}{\sqrt{2}} \leq c \leq \frac{1}{\sqrt{2}}$.

(3) $0 \leq \theta \leq \frac{\pi}{6}$






⑤ (1) $\frac{4!}{4^4} = \frac{3 \cdot \cancel{2} \cdot 1}{4 \cdot \cancel{4} \cdot 4} = \frac{3}{32}$

(2) $\frac{{}_4C_2 \times (2^5 - 2)}{4^5} = \frac{\cancel{4} \cdot 3 \cdot \cancel{(32-2)}}{\cancel{4} \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot \cancel{2}} = \frac{45}{256}$

(3) $\frac{{}_4C_1 \times 3 \times \frac{4!}{2!}}{4^5} = \frac{\cancel{4} \cdot 3 \cdot \cancel{4} \cdot 3}{\cancel{4} \cdot \cancel{4} \cdot 4 \cdot 4 \cdot 4} = \frac{9}{64}$

⑥ $y = x^{\cos x}$

$$\log y = \cos x \log x$$

$$\frac{1}{y} \frac{dy}{dx} = -\sin x \log x + \frac{1}{x} \cos x.$$

$$\frac{d}{dx}(x^{\cos x}) = x^{\cos x}\left(\frac{1}{x}\cos x - \sin x \log x\right)$$

(2). $x^{-6}+1=t$ とおく.

$$\frac{dt}{dx} = -\frac{6}{x^7}$$

$$\int \frac{4}{x^7\, t^{\frac{1}{3}}}\,dx = \int \frac{4}{\cancel{x^7}\, t^{\frac{1}{3}}} \times \left(-\frac{\cancel{x^7}}{6}\right) dt$$

$$= -\frac{2}{3}\int t^{-\frac{1}{3}}\,dt = -\frac{2}{3}\times\frac{3}{2}\times t^{\frac{2}{3}} + C$$

$$= -\left(x^{-6}+1\right)^{\frac{2}{3}} + C$$ （Cは積分定数）