①

$$f'(x) = 8 \frac{1}{\sqrt{6+\sqrt{9+x^2}}} \times \frac{1}{2} \times \frac{1}{\sqrt{6+\sqrt{9+x^2}}} \times \left(\sqrt{9+x^2}\right)'$$

$$= 4 \times \frac{1}{6+\sqrt{9+x^2}} \times \frac{1}{2} \times \frac{3x^2}{\sqrt{9+x^2}}$$

$$= \frac{6x^2}{(6+\sqrt{9+x^2})\sqrt{9+x^2}}$$

$$f'(3) = \frac{6 \times 9}{(6+6) \times 6} = \frac{3}{4}$$

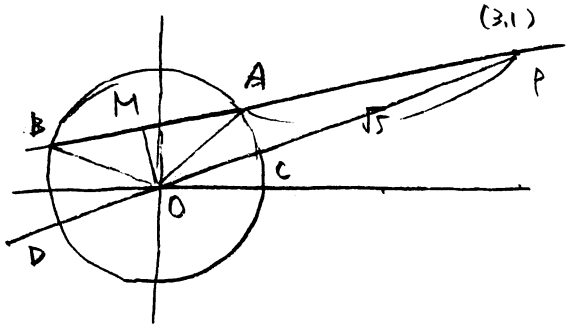
②　$(x-1)^2+(y-4)^2+(z-2)^2 = 49$　　中心 $(1, 4, 2)$　半径 7

$(-5-1)(x-1)+(6-4)(y-4)+(5-2)(z-2) = 49$

$x=y=0$ のとき　$6-8+3z-6 = 49$

$z = 19$

③



$PO = \sqrt{10}$

$PC = \sqrt{10}-1$, $PD = \sqrt{10}+1$

方べき

$PC \times PD = PA \times PB$

$10-1 = \sqrt{5} \times PB$

$PB = \frac{9}{\sqrt{5}} = \frac{9\sqrt{5}}{5}$

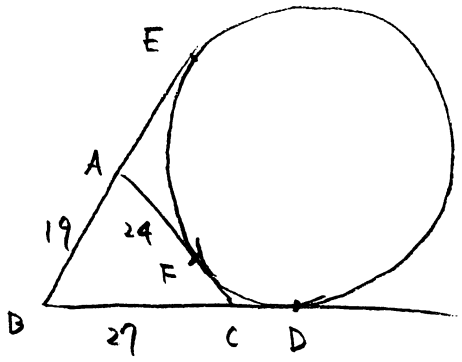
$AB = PB - PA = \frac{4\sqrt{5}}{5}$

$PM = \sqrt{PO^2 - PM^2} = \sqrt{10 - \left(\frac{7\sqrt{5}}{5}\right)^2}$

$= \sqrt{10 - \frac{49}{5}} = \frac{1}{\sqrt{5}}$

$\triangle OAB = \frac{1}{2} AB \times PM = \frac{1}{2} \times \frac{4\sqrt{5}}{5} \times \frac{1}{\sqrt{5}} = \frac{2}{5}$

④



$AF = AE$, $FC = CD$, $BE = BD$

より $BA + AE = BC + CD$

$19 + AF = 27 + FC$

$AF - FC = 8$

また $AF + FC = 24$

$\therefore AF = 16$, $FC = 8 = CD$

$BD = BC + CD = 27 + 8 = \underline{35}$

⑤ $\dfrac{4(1-\sin^2\theta) - 3}{1-\sin\theta} = \dfrac{1 - 4\sin^2\theta}{1-\sin\theta}$

$\sin\theta = t$ とおく. $\dfrac{1-4t^2}{1-t} = f(t)$ とする

$f'(t) = \dfrac{-8t(1-t) + (1-4t^2)}{(1-t)^2} = \dfrac{4t^2 - 8t + 1}{(1-t)^2}$

$4t^2 - 8t + 1 = 0$ となるのは $t = \dfrac{4 \pm \sqrt{16-4}}{4} = \dfrac{2 \pm \sqrt{3}}{2}$

$0° < \theta < 90°$ より $0 < t < 1$ なので $t = \dfrac{2-\sqrt{3}}{2}$ のみ範囲内

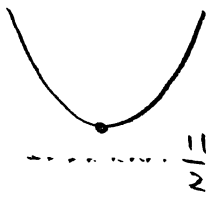
t	$0 \cdots$	$\frac{2-\sqrt{3}}{2}$	$\cdots 1$
$f'(t)$	$+$	0	$-$
$f(t)$	↗		↘

$f\left(1 - \dfrac{\sqrt{3}}{2}\right) = \dfrac{1 - 4\left(1 + \frac{3}{4} - \sqrt{3}\right)}{\frac{\sqrt{3}}{2}}$

$= \dfrac{2}{\sqrt{3}}\left(1 - 7 + 4\sqrt{3}\right)$

$= \dfrac{2\sqrt{3}}{3}\left(-6 + 4\sqrt{3}\right) = \underline{-4\sqrt{3} + 8}$

⑦



$$y = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$$

$$-\frac{b^2}{4a} + c > \frac{11}{2}$$

$$-b^2 + 4ac > 22a$$

$$2a(2c - 11) > b^2$$

$c \leq 5$ のとき 左辺 ≤ 0 なので 不可

$c = 6$ のとき $\quad 2a > b^2$.

$b = 1$. $a = 1 \sim 6$. , $b = 2$ $a = 3 \sim 6$

$b = 3$ $a = 5, 6$, $b = 4 \sim$ ×

12通り.

$$\frac{12}{216} = \frac{1}{18}$$

⑦ $x + x^{104} = (1 - x + x^2)P(x) + ax + b$.

$1 - x + x^2 = 0$ となる x を ω とおくと $\omega^2 - \omega + 1 = 0$

$\omega - 1$ を両辺にかけて $\omega^3 = -1$

$\omega + \omega^{104} = a\omega + b$

$\Leftrightarrow \omega + \omega^2 \cdot (\omega^3)^{34} = a\omega + b$

$\Leftrightarrow \omega + (\omega - 1) \cdot 1^{34} = a\omega + b$

$\Leftrightarrow 2\omega - 1 = a\omega + b$ $\quad \therefore a = 2, b = -1$

$2x - 1$

⑧ $f'(x) = \frac{2}{3x} + 4x + a$ （定義域 $x > 0$）

$g(x) = -\frac{2}{3x} - 4x$ とおくと $g'(x) = \frac{2}{3x^2} - 4 = \frac{2 - 12x^2}{3x^2}$

$g'(x) = 0$ となるのは $x = \pm \frac{1}{\sqrt{6}}$

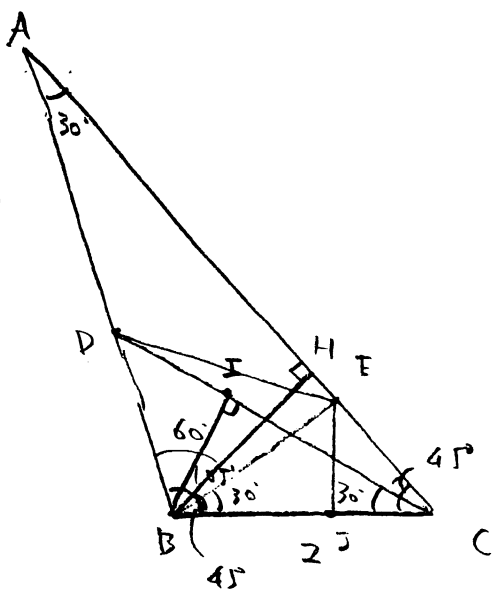
x	0 …	$\frac{1}{\sqrt{6}}$	…
$g'(x)$	+	0	−
$g(x)$	$-\infty$ ↗		↘ $-\infty$

$g\left(\frac{1}{\sqrt{6}}\right) = -\frac{2}{3}\sqrt{6} - 4 \cdot \frac{\sqrt{6}}{6}$

$= -\frac{4}{3}\sqrt{6}$

$a < -\frac{4}{3}\sqrt{6}$

⑨



Bから AC に垂線を下ろす

$CH = \sqrt{2}$, $BH = \sqrt{2}$

$AB = 2\sqrt{2}$ $AH = \sqrt{6}$

Bから CD に垂線を下ろす

$BI = 1$, $CJ = \sqrt{3}$

$BD = \sqrt{2}$, $DI = 1$

$CD = CJ + JD = \sqrt{3} + 1$

$BE = 2x$ とおく $EJ = x$, $CE = \sqrt{2}x$, $CJ = x$, $BJ = \sqrt{3}x$

$CJ + BJ = x + \sqrt{3}x = 2$ $x = \frac{2}{1+\sqrt{3}} = \sqrt{3} - 1$

$DE^2 = AD^2 + AE^2 - 2AD \cdot AE \cos 30°$

$= \sqrt{2}^2 + \left(\sqrt{6} + \sqrt{2} - \sqrt{2}(\sqrt{3} - 1)\right)^2 - 2\sqrt{2} \cdot 2\sqrt{2} \cdot \frac{\sqrt{3}}{2}$

$= 2 + 8 - 4\sqrt{3} = 10 - 4\sqrt{3}$.

⑩ $(a+1)x^2+10xy-3y^2-2ax-12y+a=0$.

$3y^2+2(6-5x)y-(a+1)x^2+2ax-a=0$

$D_1/4 = (6-5x)^2-3\{-(a+1)x^2+2ax-a\}$

$= 25x^2-60x+36+3(a+1)x^2-6ax+3a$

$= (28+3a)x^2-2(30+3a)x+36+3a$

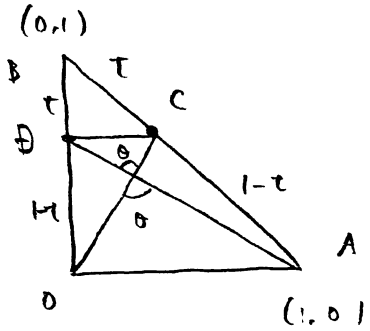
$D_2/4 = (30+3a)^2-(28+3a)(36+3a)=0$

$900+180a+9a^2-1008-192a-9a^2=0$

$12a=-108$

$a=\underline{-9}$ //

⑪



$\vec{OC} = t\begin{pmatrix}1\\0\end{pmatrix}+(1-t)\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}t\\1-t\end{pmatrix}$

$\vec{OD} = (1-t)\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}0\\1-t\end{pmatrix}$

$\vec{OC}\cdot\vec{AD} = |\vec{OC}||\vec{AD}|\cos\theta$

$\begin{pmatrix}t\\1-t\end{pmatrix}\cdot\begin{pmatrix}-1\\1-t\end{pmatrix} = \sqrt{t^2+(1-t)^2}\sqrt{1+(1-t)^2}\cos\theta$

$\cos\theta = \dfrac{-t+1-2t+t^2}{\sqrt{t^2+(1-t)^2}\sqrt{1+(1-t)^2}}$

$\cos^2\theta = \dfrac{(t^2-3t+1)^2}{(t^2+(1-t)^2)(1+(1-t)^2)}$

⑫ $Q(8-x) = -x^3 + 20x^2 - 100x + 23$

$Q = x^2 - 12x + 4 + \dfrac{9}{x-8}$

$x = 17. \quad Q = 90.$ //

⑬ $n a_1 + (n-1) a_2 + \cdots + 2a_{n-1} + 1 \cdot a_n = \dfrac{n-4}{10} + \dfrac{2}{n+5}$

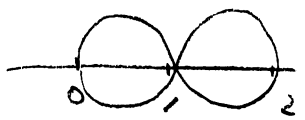
$-)\ (n-1)a_1 + (n-2)a_2 + \cdots + 1 \cdot a_{n-1} = \dfrac{n-5}{10} + \dfrac{2}{n+4}$

$a_1 + a_2 + \cdots + a_{n-1} + a_n = \dfrac{1}{10} + \dfrac{2}{n+5} - \dfrac{2}{n+4}$

$\longrightarrow \dfrac{1}{10} + 0 - 0 = \dfrac{1}{10}$ //

⑯ $y^2 = (x-1)^2 x(2-x)$

$y = \pm\sqrt{x(2-x)}\,(x-1)$



$4\int_0^1 -\sqrt{x(2-x)}\,(x-1)\,dx$

$= -4\int_0^1 \sqrt{1-(x-1)^2}\,(x-1)\,dx \qquad x-1=t \quad \dfrac{dt}{dx}=1$

$= +2\int_{-1}^{0} \sqrt{1-t^2}\,(1-t^2)'\,dt$

$= +2\left[\dfrac{2}{3}\times(1-t^2)^{\frac{3}{2}}\right]_{-1}^{0} = \dfrac{4}{3}$ //

(11) $S_x^2 = \frac{1}{100}\sum_{i=1}^{100} x_i^2 - X^2 = 5 - X^2$

$S_y^2 = \frac{1}{100}\sum_{i=1}^{100} y_i^2 - Y^2 = 9 - Y^2$

$S_{xy} = \frac{1}{100}\sum_{i=1}^{100}(x_i - X)(y_i - Y)$

$= \frac{1}{100}\sum_{i=1}^{100}(x_i y_i - X y_i - Y x_i + XY)$

$= 4XY - 3$

$\left|\frac{S_{xy}}{S_x S_y}\right| \le 1$ 이므로 $\left|\frac{4XY-3}{\sqrt{5-X^2}\sqrt{9-Y^2}}\right| \le 1$

$(4XY-3)^2 \le (5-X^2)(9-Y^2)$

$5Y^2 - 10XY + 9X^2 \le 20$

$\frac{(Y-X)^2}{4} + \frac{X^2}{5} \le 1$

$X+Y=k$ 라 하면

$5(k-2X)^2 + 4X^2 = 20$

$24X^2 - 20kX + 5k^2 - 20 = 0$

$D/4 = 100k^2 - 24(5k^2-20) \ge 0$

$100k^2 - 120k^2 + 480 \ge 0$

$k^2 \le 24$

$-2\sqrt{6} \le k \le 2\sqrt{6}$

$0 \le |X+Y| \le 2\sqrt{6}$