1

(1) (i) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta = \sin\theta(3-4\sin^2\theta)$

$A=3,\ B=4$

(ii) $3x - 4x^3 = \frac{1}{2}$.

$x=\sin\theta$ とすれば上式は $\sin 3\theta = \frac{1}{2}$ となるので $3\theta = \frac{13}{6}\pi, \frac{17}{6}\pi, \frac{25}{6}\pi$

となり $\theta = \frac{25}{18}\pi$　　$(\because \frac{3}{2}\pi \leqq 3\theta \leqq \frac{9}{2}\pi)$

(2) (i) $(\cos\frac{2}{5}\pi + i\sin\frac{2}{5}\pi)^5 = \cos 2\pi + i\sin 2\pi = 1$　∴虚部は 0

(ii) $(\alpha+i\beta)^5 = \alpha^5 + 5\alpha^4\beta i - 10\alpha^3\beta^2 - 10\alpha^2\beta^3 i + 5\alpha\beta^4 + \beta^5 i$

なので虚部は $5\alpha^4\beta - 10\alpha^2\beta^3 + \beta^5$

(iii) $\cos 5\theta + i\sin 5\theta = (\cos\theta + i\sin\theta)^5$ を展開した虚部を比較.

$\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$

$= 5(1-\sin^2\theta)^2\sin\theta - 10(1-\sin^2\theta)\sin^3\theta + \sin^5\theta$

$= 5\sin\theta - 10\sin^3\theta + 5\sin^5\theta - 10\sin^3\theta + 10\sin^5\theta + \sin^5\theta$

$= 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$

$= \sin\theta(16\sin^4\theta - 20\sin^2\theta + 5)$

$a=16,\ b=-20,\ c=5$

(iv) $f(x) = 16x^2 - 20x + 5 = 0$ を解く

$x = \frac{10 \pm \sqrt{100-80}}{16} = \frac{5\pm\sqrt{5}}{8}$

(v) $\frac{2}{5}\pi = \alpha$ とおくと $5\alpha = 2\pi$.

$\cos 3\alpha = \cos(2\pi - 2\alpha)$

$4\cos^3\alpha - 3\cos\alpha = 2\cos^2\alpha - 1$

$$4\cos^3\alpha-2\cos^2\alpha-3\cos\alpha+1=0$$

$$(\cos\alpha-1)(4\cos^2\alpha+2\cos\alpha-1)=0$$

$\cos\alpha\neq1$ だから $\cos\alpha=\dfrac{-1\pm\sqrt{5}}{4}$

$0<\alpha<\dfrac{\pi}{2}$ のとき $\cos\alpha>0$ なので $\underline{\cos\alpha=\dfrac{-1+\sqrt{5}}{4}}$

(3) $(\cos\theta+i\sin\theta)^7=\cos7\theta+i\sin7\theta$ の虚部を比較.

$$\sin7\theta={}_7C_1\cos^6\theta\sin\theta-{}_7C_3\cos^4\theta\sin^3\theta+{}_7C_5\cos^2\theta\sin^5\theta-{}_7C_7\sin^7\theta$$

$$=7(1-\sin^2\theta)^3\sin\theta-35(1-\sin^2\theta)^2\sin^3\theta+21(1-\sin^2\theta)\sin^5\theta-\sin^7\theta$$

$$=7\sin\theta-21\sin^3\theta+\underline{21\sin^5\theta}\underline{-7\sin^7\theta}-35\sin^3\theta+\underline{70\sin^5\theta}\underline{-35\sin^7\theta}$$

$$\underline{+21\sin^5\theta}\underline{-21\sin^7\theta}\underline{-\sin^7\theta}$$

$$=-64\sin^7\theta+112\sin^5\theta-56\sin^3\theta+7\sin\theta$$

$$=\sin\theta\left(-64\sin^6\theta+112\sin^4\theta-56\sin^2\theta+7\right)\cdots①$$

$p=-64,\ q=112,\ r=-56,\ s=7$

-1　7　14　7

-32　28　8　7

$\sin\left(\dfrac{2\pi}{7}\times k\right)=z_k\ (k=0,1,2\ldots6)$ と表す.

$\sin\left(\dfrac{2\pi}{7}k\times7\right)=0$ だから. $z_1, z_2\ldots z_6$ は $-64x^6+112x^4-56x^2+7=0$ を満たす. また. $z_1=-z_6,\ z_2=-z_5,\ z_3=-z_4$ なので.

$g(x)=0$ の3つの解は $z_1^2,\ z_2^2,\ z_3^2$ となっている.

解と係数の関係より $z_1^2\times z_2^2\times z_3^2=\dfrac{7}{64}$

$z_1, z_2, z_3>0$ だから $z_1\times z_2\times z_3=\dfrac{\sqrt{7}}{8}$

$\therefore\ \sin\dfrac{2\pi}{7}\times\sin\dfrac{4}{7}\pi\times\sin\dfrac{6}{7}\pi=\underline{\dfrac{\sqrt{7}}{8}}$

$\boxed{2}$ (1)

(i) $\left|2+\frac{3}{2}i\right|=\sqrt{2^2+\left(\frac{3}{2}\right)^2}=\underline{\frac{5}{2}}$ //

(ii) $\left|z-2-\frac{3}{2}i\right|=\frac{5}{2}\sqrt{2}$, $z=\frac{1}{w}$. を連立.

$$\left|\frac{1}{w}-2-\frac{3}{2}i\right|=\frac{5}{2}\sqrt{2}$$

$$\left|\frac{1-\left(2+\frac{3}{2}i\right)w}{w}\right|=\frac{5}{2}\sqrt{2}$$

$$\left|\left(2+\frac{3}{2}i\right)\left(w-\frac{1}{2+\frac{3}{2}i}\right)\right|=\frac{5}{2}\sqrt{2}|w|$$

$$\frac{5}{2}\left|w-\frac{2-\frac{3}{2}i}{4+\frac{9}{4}}\right|=\frac{5}{2}\sqrt{2}|w|$$

$$\left|w-\underline{\frac{8-6i}{25}}\right|=\sqrt{2}|w| \quad //$$

$w=x+yi$ とすると.

$$\left|x-\frac{8}{25}+\left(y+\frac{6}{25}\right)i\right|=\sqrt{2}|x+yi|$$

$$\left(x-\frac{8}{25}\right)^2+\left(y+\frac{6}{25}\right)^2=2(x^2+y^2)$$

$$x^2+\frac{16}{25}x-\frac{8^2}{25^2}+y^2-\frac{12}{25}y-\frac{6^2}{25^2}=0$$

$$\left(x+\frac{8}{25}\right)^2+\left(y-\frac{6}{25}\right)^2=\frac{200}{25^2}=\frac{8}{25}=\left(\frac{2\sqrt{2}}{5}\right)^2$$

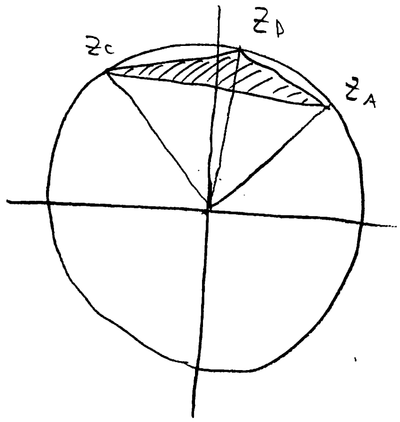
中心 $\underline{-\frac{8}{25}+\frac{6}{25}i}$ // 半径 $\underline{\frac{2}{5}\sqrt{2}}$,

(2) $\frac{z_B}{z_A}=\cos\theta+i\sin\theta=\cos\left(\pm\frac{2}{3}\pi\right)+i\sin\left(\pm\frac{2}{3}\pi\right)$

$$\theta=\underline{\frac{4}{3}\pi} \quad //$$

$$\frac{z_C}{z_A} = \cos 2\theta + i\sin 2\theta = \cos(\pm\pi) + i\sin(\pm\pi)$$

$$2\theta = 3\pi \qquad \theta = \frac{3}{2}\pi$$

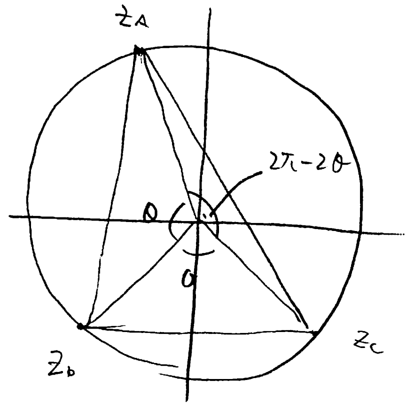


(ii) $0 < \theta \leqq \frac{\pi}{2}$ のとき

$$S = \frac{1}{2}\times 1\times 1\times \sin\theta \times 2 - \frac{1}{2}\times 1\times 1\times \sin 2\theta$$

$$= \sin\theta - \frac{1}{2}\sin 2\theta$$

$\frac{\pi}{2} < \theta < \pi$ のとき



$$S = \frac{1}{2}\times 1\times 1\times \sin\theta\times 2 + \frac{1}{2}\times 1\times 1\times \sin(2\pi-2\theta)$$

$$= \sin\theta - \frac{1}{2}\sin 2\theta$$

$$\therefore S = \sin\theta - \frac{1}{2}\sin 2\theta$$

$$\therefore S = \sin\theta - \frac{1}{2}\sin 2\theta$$

$$\frac{dS}{d\theta} = \frac{1}{2}\cos\theta - \cos 2\theta = \frac{1}{2}\cos\theta - 2\cos^2\theta + 1 = 0 \text{ より}$$

$(\cos\theta - 1)(2\cos\theta + 1) = 0.$ $\quad \cos\theta = 1, -\frac{1}{2}$

$$\theta = \frac{2}{3}\pi$$

増減は右のようになるので、

θ	0	$\cdots$	$\frac{2}{3}\pi$	$\cdots$	π
S'		$+$	0	$-$	
S		↗		↘	

$\theta = \frac{2}{3}\pi$ のとき S は最大で $S = \sin\frac{2}{3}\pi - \frac{1}{2}\sin\frac{4}{3}\pi = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4} = \frac{3}{4}\sqrt{3}$

(iii) $1 = 2m - 3n$

$\underline{1 = 2\times 2 - 3\times 1 \quad (-}$

$2(m-2) - 3(n-1) = 0$

$m-2 = 3k, \quad n-1 = 2k.$

$m = 2+3k, \quad n = 1+2k$

$50 \leqq 2+3k \leqq 150$ かつ $50 \leqq 1+2k \leqq 150$

$\Leftrightarrow 25 \leqq k \leqq 49 \qquad k$は25個

3

(1) $f'(x) = 1 + \frac{2}{\pi} \times \pi \cos \pi x = 1 + 2\cos \pi x$

$f'(x) = 0$ となるのは $\cos \pi x = -\frac{1}{2}$ なので $x = \frac{2}{3}$

x	0	…	$\frac{2}{3}$	…	1
f'		+	0	−	
f		↗		↘	

$x = \frac{2}{3}$ のとき最大

$f\left(\frac{2}{3}\right) = \frac{2}{3} + \frac{2}{\pi} \times \sin \frac{2}{3}\pi$

$= \frac{2}{3} + \frac{\sqrt{3}}{\pi}$

(2)

(i) l_t : $y = (1 + 2\cos \pi t)(x - t) + t + \frac{2}{\pi} \sin(\pi t)$

$t = 0$ のとき l_0 : $y = 3x$

$t = 1$ のとき l_1 : $y = -1(x-1) + 1 + 0 = -x + 2$

(ii) l_t の y 切片は $-t - 2t\cos \pi t + t + \frac{2}{\pi}\sin(\pi t) = \frac{2}{\pi}\sin(\pi t) - 2t\cos \pi t$.

これを $g(t)$ とおくと

$g'(t) = 2\cos \pi t - 2\cos \pi t + 2t\pi \sin \pi t = 2\pi t \sin(\pi t)$

$g'(t) = 0$ となるのは $t = 0, 1$.

$t = 0$ のとき最小 $g(0) = 0$ (l_0)

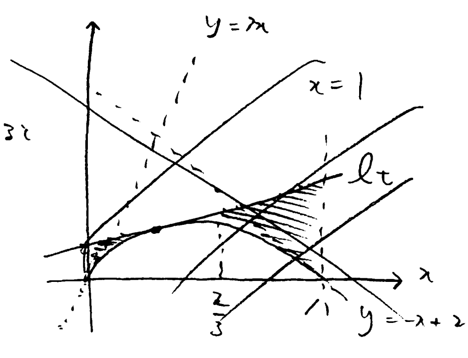
$t = 1$ のとき最大 $g(1) = 2$. (l_1)

t	0	…	1
g'	0	+	0
g		↗	

(iii) $f(0) = 0$. $f(1) = 1$.

$y = f(x)$ と x 軸で囲まれる図形の面積を S_0 とすると

S は l_t, $x = 0$, $x = 1$, $y = 0$ で囲まれる図形の面積から S_0 を引いたものに等しく、また S_0 は t の値とは無関係な定数である



l_t と $x=1$ の交点の y 座標は $1+2\cos\pi t - t - 2t\cos\pi t + \frac{2}{\pi}\sin\pi t$

台形の面積は $\frac{1}{2}\left(1+2\cos\pi t - t - 2t\cos\pi t + \frac{2}{\pi}\sin\pi t + \frac{2}{\pi}\sin(\pi t) - 2\cos\pi t\right)\times 1$

$$= \frac{1}{2}\left(1 - t - 2t\cos\pi t + \frac{4}{\pi}\sin\pi t \cancel{+ 2\cos\pi t + 4t\cos\pi t}\right)$$

$$S_0 = \int_0^1 x + \frac{2}{\pi}\sin\pi x\,dx = \left[\frac{1}{2}x^2 - \frac{2}{\pi^2}\cos\pi x\right]_0^1 = \frac{1}{2} + \frac{2}{\pi^2} + \frac{2}{\pi^2} = \frac{1}{2}+\frac{4}{\pi^2}$$

$$h'(t) = -\frac{1}{2} + \cos\pi t - \pi t\sin\pi t \cancel{+ 2\cos\pi t} - \pi\sin\pi t \cancel{- 2\cos\pi t} + 2\pi t\sin\pi t$$

$$\left(\frac{2}{\pi}\sin(\pi t) - 2t\cos\pi t + (1+2\cos\pi t)(1-t) + t + \frac{2}{\pi}\sin(\pi t)\right)\times 1 \times \frac{1}{2}$$

$$= \underline{\frac{1}{\pi}\sin\pi t} - \underline{t\cos\pi t} + \frac{1}{2} - \cancel{\frac{1}{2}t} + \underline{\cos\pi t} - \underline{t\cos\pi t} + \cancel{\frac{1}{2}t} + \underline{\frac{1}{\pi}\sin\pi t}$$

$$= \frac{2}{\pi}\sin\pi t - 2t\cos\pi t + \cos\pi t + \frac{1}{2} = h(t)$$

$$h'(t) = \cancel{2\cos\pi t} - \cancel{2\cos\pi t} + 2t\pi\sin\pi t - \pi\sin\pi t$$

$$= \pi\sin\pi t\,(2t-1)$$

$t=\frac{1}{2}$ のとき最小、

$$h\left(\tfrac{1}{2}\right) = \frac{2}{\pi}\sin\frac{\pi}{2} - 2\times\frac{1}{2}\cos\frac{\pi}{2} + \cos\frac{\pi}{2} + \frac{1}{2} = \frac{2}{\pi} + \frac{1}{2}$$

$$S = \frac{2}{\pi} + \frac{1}{2} - \frac{1}{2} - \frac{4}{\pi^2} = \underline{\frac{2}{\pi} - \frac{4}{\pi^2}}$$