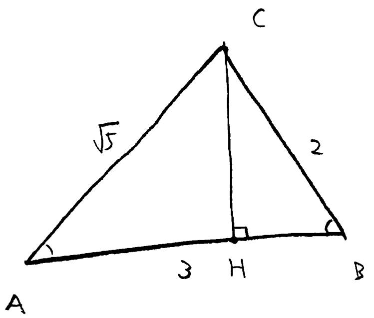
①



$$\cos\angle A = \frac{9+5-4}{2\cdot 3\cdot\sqrt{5}} = \frac{\cancel{10}^{\,5}}{{}_{3}\cancel{6}\sqrt{5}} = \frac{\sqrt{5}}{3}$$

$$AH = AC \times \cos\angle A = \frac{5}{3}$$

$$CH = \sqrt{\sqrt{5}^2 - \left(\frac{5}{3}\right)^2} = \frac{2\sqrt{5}}{3}$$

$$\frac{1}{\tan A} + \frac{1}{\tan B} = \frac{AH}{CH} + \frac{BH}{CH}$$

$$= \frac{AH + BH}{\frac{2\sqrt{5}}{3}} = \frac{9}{2\sqrt{5}} = \frac{9\sqrt{5}}{10}$$

②

$$\frac{a+bx}{1-x+x^2} + \frac{c}{1+x} = \frac{a+ax+bx+bx^2+c-cx+cx^2}{(1-x+x^2)(1+x)} = \frac{(b+c)x^2+(a+b-c)x+a+c}{1+x^3}$$

$b+c=0,\quad a+b-c=-1,\quad a+c=1$

$\Rightarrow\quad a+2b=-1 \qquad 2a+b=0 \qquad \Rightarrow\quad a=\frac{1}{3},\ b=-\frac{2}{3},\ c=\frac{2}{3}$

$$\int_0^1 \frac{\frac{1}{3}-\frac{2}{3}x}{1-x+x^2}\,dx + \int_0^1 \frac{\frac{2}{3}}{1+x}\,dx$$

$$= -\frac{1}{3}\int_0^1 \frac{(1-x+x^2)'}{1-x+x^2}\,dx + \frac{2}{3}\int_0^1 \frac{1}{1+x}\,dx$$

$$= -\frac{1}{3}\Big[\log|1-x+x^2|\Big]_0^1 + \frac{2}{3}\Big[\log|1+x|\Big]_0^1$$

$$= 0 + \frac{2}{3}\log 2 = \frac{2}{3}\log 2$$

③

$2^x = t$ とする $(t>0)$

$$t - 2^8 \leqq 4 - 2^{10}\frac{1}{t}$$

$$\Leftrightarrow \quad t^2 - (2^8+4)t + 2^{10} \leqq 0$$

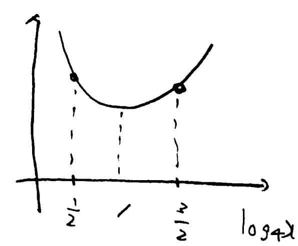
$$\Leftrightarrow \quad (t-2^8)(t-2^2) \leqq 0$$

$$2^2 \leqq t \leqq 2^8$$

$$2^2 \leqq 2^x \leqq 2^8 \qquad \therefore \quad \underline{2 \leqq x \leqq 8}$$

$$f(x) = \log_4 x + \frac{1}{\log_4 x}$$

ここで $2 \leqq x \leqq 8$ より $\frac{1}{2} \leqq \log_4 x \leqq \frac{3}{2}$



$$f(2) = \frac{1}{2} + \frac{1}{\frac{1}{2}} = \frac{5}{2}, \quad f(8) = \frac{3}{2} + \frac{1}{\frac{3}{2}} = \frac{13}{6} < \frac{5}{2}$$

$$f(1) = 1 + 1 = 2.$$

<u>最小値は 2，最大値は $\frac{5}{2}$</u>

④

$$r = -16\sin\theta\cos\frac{\pi}{3} - 16\cos\theta\sin\frac{\pi}{3}$$

$$= -8\sin\theta - 8\sqrt{3}\cos\theta$$

$$= -8\frac{y}{r} - 8\sqrt{3}\frac{x}{r}$$

$$r^2 = -8y - 8\sqrt{3}x = x^2 + y^2$$

$$(x+4\sqrt{3})^2 + (y+4)^2 = 64$$

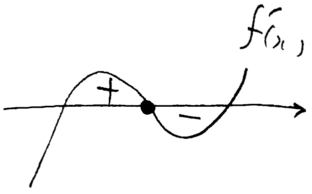
<u>中心 $(-4\sqrt{3}, -4)$ 半径 8 の円</u>

⑤

$f'(x) = 4x^3 - 12x^2 + a$

$f''(x) = 12x^2 - 24x = 12x(x-2)$ $\underline{0,\ 2}$

極大値をもつのは $f'(x)=0$ となる x があり、その前後で符号が正から負に転じるとき。(右グラフ)



そのためには $f'(0)>0$ かつ $f'(2)<0$ が必要十分となる

$f'(0) = a > 0$, $f'(2) = 32 - 48 + a < 0$

$\therefore\ \underline{0 < a < 16}$

⑥

$\overrightarrow{OA} = (5\cos\alpha, 5\sin\alpha)$, $\overrightarrow{OB} = (3\cos\beta, 3\sin\beta)$ とおく。

$$\begin{cases} 5\cos\alpha + 3\cos\beta = 2\times 3 & \cdots ① \\ 5\sin\alpha + 3\sin\beta = \sqrt{2}\times 3 & \cdots ② \end{cases}$$

$\overrightarrow{OA}\cdot\overrightarrow{OB} = 15\cos\alpha\cos\beta + 15\sin\alpha\sin\beta$

$①^2 + ②^2$

$25 + 9 + 30(\cos\alpha\cos\beta + \sin\alpha\sin\beta) = 36 + 18$

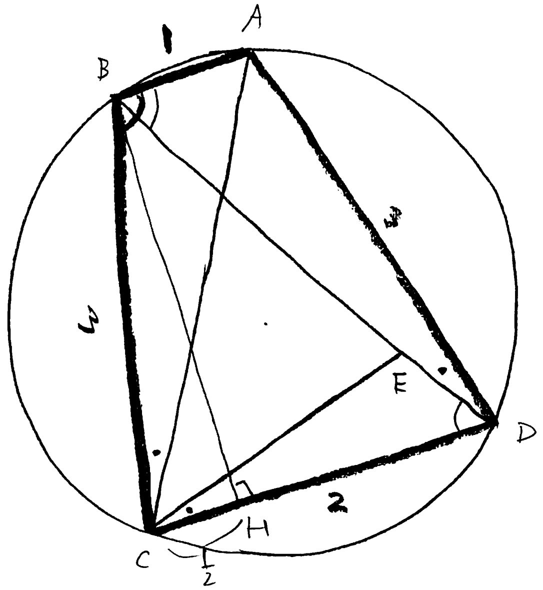
$\cos\alpha\cos\beta + \sin\alpha\sin\beta = \frac{2}{3}$

$\therefore\ \overrightarrow{OA}\cdot\overrightarrow{OB} = \underline{10}$

$\triangle OAB = \frac{1}{2}\sqrt{|\overrightarrow{OA}|^2|\overrightarrow{OB}|^2 - (\overrightarrow{OA}\cdot\overrightarrow{OB})^2} = \frac{1}{2}\sqrt{25\times 9 - 10^2}$

$= \frac{5}{2}\sqrt{9-4} = \underline{\frac{5\sqrt{5}}{2}}$

⑦



ABCDは等脚台形。BからCDに下ろした垂線の足をHとすると、

$CH = (CD - 1) \div 2 = \frac{1}{2}$.

$\cos\angle BCH = \frac{\frac{1}{2}}{3} = \frac{1}{6}$

$\cos\angle ABC = \cos(180° - \angle BCH) = \underline{-\frac{1}{6}}$

$AC^2 = 1^2 + 3^2 - 2\cdot 1\cdot 3\cdot \cos\angle ABC = 11 \qquad AC = \sqrt{11}$

$\triangle ABC \equiv \triangle BAD$

$\angle ABD = \angle EDC$, $\angle ADB = \angle ECD$ だから

$\triangle ABD \sim \triangle EDC$

$AD : DB = EC : CD$

$3 : \sqrt{11} = EC : 2$

$EC = \frac{6}{\sqrt{11}} = \underline{\frac{6\sqrt{11}}{11}}$

⑧

A列 $a_1, a_2 \dots a_{15}$　　B列 $b_1, b_2 \dots b_{10}$.

$\frac{1}{15}\sum_{k=1}^{15} a_k = 70$,　　$\frac{1}{15}\sum_{k=1}^{15} a_k^2 - 70^2 = 10$

$\frac{1}{10}\sum_{k=1}^{10} b_k = 80$　　$\frac{1}{10}\sum_{k=1}^{10} b_k^2 - 80^2 = 15$

$$\frac{1}{25}\left(\sum_{k=1}^{15} a_k + \sum_{k=1}^{10} b_k\right) = \frac{1}{25}(70\times15 + 80\times10) = \frac{\overset{370}{\cancel{1850}}}{\underset{5}{\cancel{25}}} = \underline{74}$$

$$\frac{1}{25}\left(\sum_{k=1}^{15} a_k^2 + \sum_{k=1}^{10} b_k^2\right) - 74^2 = \frac{1}{25}\Big((10+4900)\times15 + (15+6400)\times10\Big) - 74^2$$

$$= \frac{1}{5}(4910\times3 + 6415\times2) - 74^2 = 982\times3 + 1283\times2 - 74^2$$

$$= 2946 + 2566 - 5476 = \underline{36}$$

⑨

$$2a_{n+1} + b_{n+1} = \frac{2a_n - 2b_n}{3} \cancel{-1} + \frac{2a_n + 4b_n}{3} \cancel{+1}$$

$$= \frac{2}{3}(2a_n + b_n)$$　　公比 $\underline{\frac{2}{3}}$

$$2a_1 + b_1 = \frac{4}{3} + \frac{1}{4} = \frac{19}{12}$$

$$2a_n + b_n = \frac{19}{12}\times\left(\frac{2}{3}\right)^{n-1}$$　　$$a_n = -\frac{1}{2}b_n + \frac{19}{24}\left(\frac{2}{3}\right)^{n-1}$$

$$b_{n+1} = \frac{2}{3}\left(-\frac{1}{2}b_n + \frac{19}{24}\left(\frac{2}{3}\right)^{n-1}\right) + \frac{4}{3}b_n + 1 = b_n + \frac{19}{36}\left(\frac{2}{3}\right)^{n-1} + 1$$

$$b_n = b_1 + \sum_{k=1}^{n-1}\left\{\frac{19}{36}\left(\frac{2}{3}\right)^{k-1} + 1\right\} = b_1 + n - 1 + \frac{19}{36}\times\frac{1-\left(\frac{2}{3}\right)^{n-1}}{1-\frac{2}{3}}$$

$$b_n - n = \frac{1}{4} - 1 + \frac{19}{12}\left(1 - \left(\frac{2}{3}\right)^{n-1}\right) \to \frac{1}{4} - 1 + \frac{19}{12} = \underline{\frac{5}{6}}$$

⑩

$104 = 2^3 \times 13$　　$3 \leqq a+b+c \leqq 27$, $2 \leqq d+e \leqq 18$ に注意して.

$(a+b+c,\ d+e) = (8, 13), (13, 8), (26, 4)$

(i) $\underline{a+b+c=8}$, $\underline{d+e=13}$

$\to (d, e) = (4, 9), (5, 8), (6, 7)$

$\to (a, b, c) = (1, 1, 6), (1, 2, 5)(1, 3, 4), (2, 2, 4), (2, 3, 3)$

$3 \times 5 = 15$

(ii) $\underline{a+b+c=13}$, $d+e=8$

$(d, e) = (1, 7), (2, 6), (3, 5), (4, 4)$

$\to (a, b, c) = (1, 3, 9), (1, 4, 8), (1, 5, 7), (1, 6, 6)$

$, (2, 2, 9), (2, 3, 8), (2, 4, 7), (2, 5, 6)$

$, (3, 3, 7), (3, 4, 6), (3, 5, 5), (4, 4, 5)$

$12 \times 4 = 48$

(iii) $\underline{a+b+c=26}$, $d+e=4$

$(d, e) = (1, 3), (2, 2)$

$\to (a, b, c) = (8, 9, 9)$　　$2 \times 1 = 2$

$15 + 48 + 2 = \underline{65\text{通り}}$

$a \leqq b \leqq c \leqq d \leqq e$ を満たすのは

(i) $(d, e) = (4, 9)$ のとき　3つ.

$(d, e) = (5, 8)$　〃　4つ

$(d, e) = (6, 7)$　〃　5つ

(ii)(iii) のときは. $c \leqq d$ とはなりえない.

$\underline{12\text{通り}}$