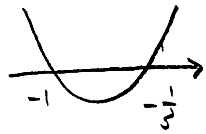
①

a	0	1	2	3	4
$P(a)$	$\frac{1}{2^4}$	$\frac{4}{2^4}$	$\frac{6}{2^4}$	$\frac{4}{2^4}$	$\frac{1}{2^4}$

$a+c=4$



(1) $f(-1) = a-b+c = 0$

$f(-\frac{1}{3}) = \frac{1}{9}a - \frac{1}{3}b + c = 0 \iff a-3b+9c=0$

$b=4.$

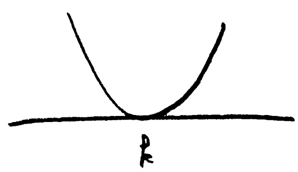
$$\begin{array}{rl} & a+9c = 12 \\ -) & a+c = 4 \\ \hline & 8c = 8 \end{array}$$

$c=1,\quad a=3$

$(a,b,c) = \underline{(3,4,1)}$ //

$\frac{4}{2^4} \times \frac{1}{6} = \underline{\frac{1}{24}}$ //

(2) $b^2 - 4ac = 0 \qquad c = 4-a.$



$b^2 - 4a(4-a) = 0$

$4a^2 - 16a + b^2 = 0$

$4(a-2)^2 + b^2 = 16$

$b = \cancel{1}, \cancel{2}, \cancel{3}, \boxed{4} \qquad a=2 \qquad c=2.$

$\frac{6}{2^4} \times \frac{1}{6} = \underline{\frac{1}{16}}$ //

$2x^2+4x+2=0 \iff (x+1)^2=0 \qquad x = \underline{-1}$ //

(3) $D = b^2 - 4ac = 4(a-2)^2 + b^2 - 16 < 0$

$4(a-2)^2 + b^2 < 16$

$a=1 \quad b^2<12 \quad b=1,2,3$

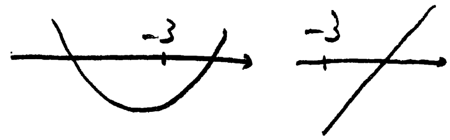
$a=2 \quad b^2<16 \quad b=1,2,3$

$a=3 \quad b^2<12 \quad b=1,2,3$

$$\frac{3}{6} \times \frac{4+6+4}{2^4} = \frac{7}{16}$$

$$f(-3) = 9a - 3b + c$$

$$= 8a - 3b + 4 \leqq 0$$



$$8a + 4 \leqq 3b$$

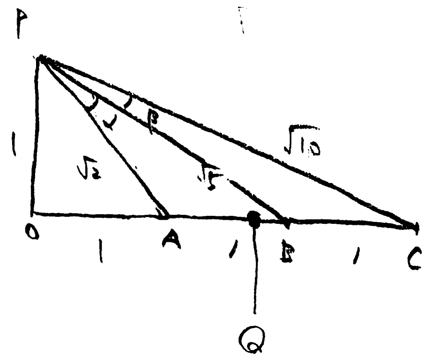
$a = 0$. $\quad b \geqq \frac{4}{3} \quad b = 2, 3, 4, 5, 6$

$a = 1 \quad b \geqq 4 \quad b = 4, 5, 6$

$a = 2 \quad b \geqq \frac{20}{3} \quad \times$

$$\frac{1}{2^4} \times \frac{5}{6} + \frac{4}{2^4} \times \frac{3}{6} = \frac{17}{2^2 \times 6} = \frac{17}{96}$$

②



$$\cos\alpha = \frac{2+5-1}{2\cdot\sqrt{2}\sqrt{5}} = \frac{3}{\sqrt{10}}$$

$$\cos\beta = \frac{5+10-1}{2\cdot\sqrt{5}\sqrt{10}} = \frac{14}{10\sqrt{2}} = \frac{7}{5\sqrt{2}}$$

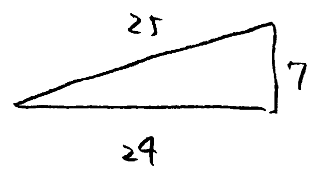
$$\cos 2\beta = 2\cos^2\beta - 1 = \frac{49}{50}\times 2 - 1 = \frac{48}{50} = \underline{\frac{24}{25}}$$

$$\cos^2\alpha = \frac{9}{10} = \frac{1125}{1250} \qquad \cos^2 2\beta = \frac{24^2}{25^2} = \frac{1152}{1250}$$

(2) $24^2 = 576$ $\quad 625 - 576 = 49$

$$\cos\alpha < \cos 2\beta$$

$$\alpha > 2\beta$$

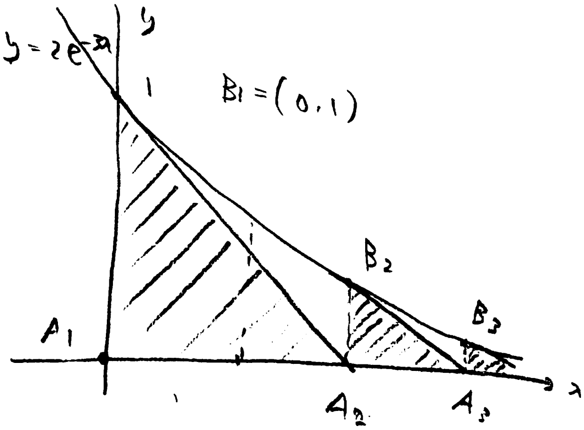


$$\tan\angle OPQ = \tan\left(\frac{\pi}{4} + 2\beta\right) = \frac{1+\tan 2\beta}{1-\tan 2\beta}$$

$$= \frac{1+\frac{7}{24}}{1-\frac{7}{24}} = \underline{\frac{31}{17}}$$

$$1 + 1 - \frac{31}{17} = \underline{\frac{3}{17}}$$

③



$$y' = 2 \cdot (-3) e^{-3x} = -6e^{-3x}$$

$$y = -6e^{-3a_n}(x - a_n) + 2e^{-3a_n}$$

$$= -6e^{-3a_n}x + (6a_n + 2)e^{-3a_n} \qquad a_1 = 0$$

$y = 0$ のとき $\qquad 6e^{-3a_n} x = (6a_n + 2)e^{-3a_n}$

$$x = a_n + \frac{1}{3} = a_{n+1}$$

$$a_{n+1} - a_n = \frac{1}{3} \qquad a_n = 0 + (n-1) \times \frac{1}{3} = \underline{\frac{n-1}{3}}$$

$$S_n = \frac{1}{2}(a_2 - a_1) \times 2e^{-3a_1} + \frac{1}{2}(a_3 - a_2) \times 2e^{-3a_2} + \cdots + \frac{1}{2}(a_{n+1} - a_n) 2e^{-3a_n}$$

$$= \frac{1}{3}e^{0} + \frac{1}{3}e^{-1} + \frac{1}{3}e^{-2} + \cdots + \frac{1}{3}e^{1-n}$$

$$= \frac{1}{3} \times \frac{1 - \left(\frac{1}{e}\right)^n}{1 - \frac{1}{e}} \longrightarrow \underline{\frac{1}{3} \times \frac{e}{e-1}}$$

$$T_n = \int_0^{a_{n+1}} 2e^{-3x}\,dx = \left[-\frac{2}{3}e^{-3x} \right]_0^{\frac{n}{3}} = -\frac{2}{3}e^{-n} + \frac{2}{3}$$

$$= \frac{2}{3}(1 - e^{-n})$$

$$\frac{T_n}{S_n} = \frac{2}{\cancel{3}} \times \frac{e^n - 1}{e^n} \times \frac{\cancel{3}\left(1 - \frac{1}{e}\right)}{1 - \left(\frac{1}{e}\right)^n} = 2 \times \cancel{\frac{e^n - 1}{e^n}} \times \frac{\cancel{e^n}(e-1)}{\cancel{(e^n - 1)}e}$$

$$= \frac{2(e-1)}{e} = \underline{2\left(1 - \frac{1}{e}\right)}$$

④

$C_2: \quad 2x = -y^2 + 6y$

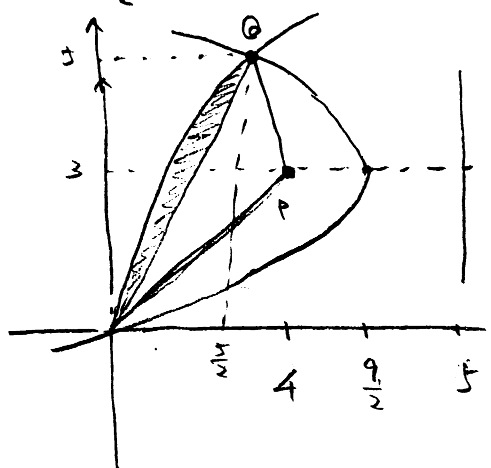
$4 \cdot \frac{1}{2} \cdot x = -(y-3)^2 + 9$

$4 \cdot \frac{1}{2}\left(x - \frac{9}{2}\right) = -(y-3)^2$

焦点は $\left(-\frac{1}{2} + \frac{9}{2},\ 3\right) = \underline{(4, 3)}$

準線は $x = +\frac{1}{2} + \frac{9}{2} = 5$.

頂点は $(x, y) = \left(\frac{9}{2}, 3\right)$



$10x + 2x - 6\sqrt{10x} = 0$

$2x = \sqrt{10x}$

$4x^2 = 10x$

$x = 0,\ \frac{10}{4} = \frac{5}{2}$

$(x, y) = \underline{\left(\frac{5}{2}, 5\right)} \quad \cdots Q$

$P(4, 3) \quad Q\left(\frac{5}{2}, 5\right)$

$y = \frac{3-5}{4-\frac{5}{2}}(x-4)+3$

$= \frac{-2 \times 2}{3}(x-4)+3$

$3y = -4x + 16 + 9$

$\underline{4x + 3y = 25}$

$\triangle OPQ = \frac{1}{2}\left|4 \times 5 - 3 \times \frac{5}{2}\right|$

$= \frac{1}{2}\left|20 - \frac{15}{2}\right|$

$= \underline{\frac{25}{4}}$

$V = \pi\left(\frac{5}{2}\right)^2 \times 5 \times \frac{1}{3} - \int_0^5 \pi x^2\,dy = \frac{125}{12}\pi - \int_0^5 \frac{\pi}{100}y^4\,dy$

$= \frac{125}{12}\pi - \frac{\pi}{500}5^5 = \frac{125}{12}\pi - \frac{25}{4}\pi = \underline{\frac{25}{6}\pi}$