①

(2)

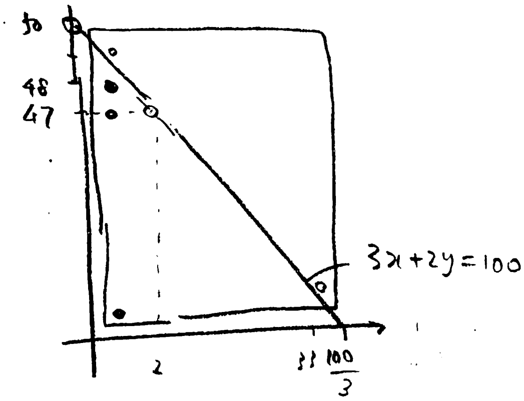
$$\begin{array}{r} 3x + 2y = 100 \\ -)\ 3\cdot 2 + 2\cdot 47 = 100 \\ \hline 3(x-2) + 2(y-47) = 0 \end{array}$$

$3(x-2) = -2(y-47) = 6n$

$x = 2n+2$　　$x>0$ より　$2n>2$　　$n>-1$

$y = -3n+47$　　$y>0$ より　$-3n+47>0$　　$n < \frac{47}{3} < 16$

$0 \leqq n \leqq 15$　　<u>16個</u>



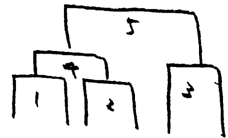
$(33 \times 49 - 16) \div 2$

$= 808$

②

(1) ${}_6C_2 \times {}_4C_2 \times {}_2C_2 = 15 \times 6 \times 1 = \underline{90}$ (ア)

$90 \times 2^4 = \underline{1440}$ (イ)



(2) $P(A_1) = \dfrac{{}_4C_2}{90} = \underline{\dfrac{1}{15}}$ (ウ).(エ)

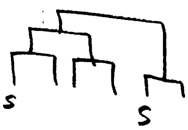
$P(A_1 \cap B) = \dfrac{1}{15} \times \dfrac{2}{3} \times \dfrac{2}{3} = \underline{\dfrac{4}{135}}$ (オ).(カ)

(3) $P(A_2 \cap B) = \dfrac{4}{135}$ $\quad P(A_3 \cap B) = \dfrac{{}_4C_2}{90} \times \dfrac{2}{3} = \dfrac{2}{45}$

$P(A_4 \cap B) = \dfrac{{}_2C_1 \times {}_4C_1 \times {}_1C_1 \times {}_3C_1}{90} \times \dfrac{2}{3} \times \dfrac{2}{3} \times 1 \times \dfrac{2}{3} = \dfrac{4}{15} \times \dfrac{8}{27} = \dfrac{32}{405}$

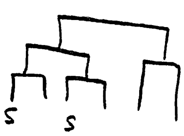
$P(A_5 \cap B) = \dfrac{{}_2C_1 \times {}_4C_1 \times {}_3C_2 \times {}_1C_1 \times {}_1C_1 \times 2}{90} \times \dfrac{2}{3} \times \dfrac{2}{3} \times 1 = \dfrac{8}{15} \times \dfrac{4}{9} = \dfrac{32}{135}$

どちらかは負けて対戦はないとき



$\dfrac{{}_2C_1 \times {}_4C_1 \times {}_3C_2}{90} \times \left(\dfrac{1}{3} \times \dfrac{2}{3} \times \dfrac{2}{3} + \dfrac{2}{3} \times \dfrac{1}{3} \times \dfrac{2}{3} \times \dfrac{2}{3} + \dfrac{2}{3} \times \dfrac{1}{3} \times \dfrac{2}{3} \times \dfrac{2}{3} \right) \times 2.$

$= \dfrac{4}{15} \times \dfrac{1}{3^4} (12 + 8 + 8) \times 2 = \dfrac{224}{1215}$



$\dfrac{{}_2C_1 \times {}_4C_1 \times {}_1C_1 \times {}_3C_1}{90} \times \left(\dfrac{2}{3} \times \dfrac{1}{3} \times \dfrac{2}{3} \times \dfrac{2}{3} \times 2 \right)$

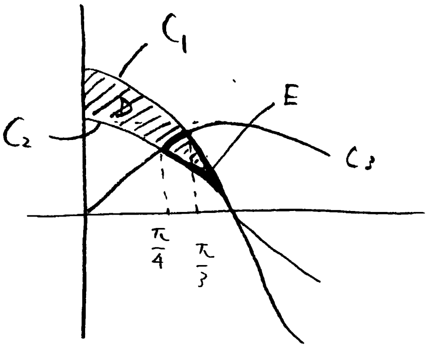
$= \dfrac{4}{15} \times \dfrac{16}{81} = \dfrac{64}{1215}$

$\dfrac{1}{1215} (36 \times 2 + 54 + 96 + 288 + 224 + 64) = \dfrac{798}{1215} = \dfrac{202}{405}$

$\dfrac{512}{1215}$

③

(1)



$$\int_0^{\frac{\pi}{2}} \sqrt{3}\cos x - \cos x \, dx$$

$$= (\sqrt{3}-1)\left[\sin x\right]_0^{\frac{\pi}{2}} = \sqrt{3}-1.$$

$$\sqrt{3}\cos t = \sin t$$

$$\cos^2 t + 3\cos^2 t = 1$$

$$\cos t = \frac{1}{2}$$

$$S_2 = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sin x \, dx + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sqrt{3}\cos x \, dx - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x \, dx$$

$$= -\left[\cos x\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} - \sqrt{3}\left[\sin x\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} - \left[\sin x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

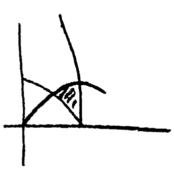
$$= \sqrt{3} + \sqrt{2} - 3$$

(2)

$$V_2 = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \pi \sin^2 x \, dx + \sqrt{3}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \pi \cos^2 x \, dx - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \pi \cos^2 x \, dx$$

$$= \frac{\pi(\pi + 3 - 3\sqrt{3})}{6}.$$

(3)



$$\lim_{a \to \infty} S_2 = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x - \cos x \, dx = \left[-\cos x - \sin x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= -1 - 0 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2} - 1$$

(4)