[1]　(1) 解と係数の関係より

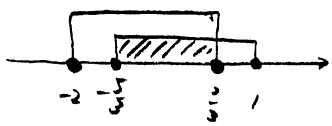
$$\alpha+\beta=-\frac{5}{3},\quad \alpha\beta=\frac{8}{3}$$

$$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=\left(-\frac{5}{3}\right)^2-2\times\frac{8}{3}=\underline{-\frac{23}{9}}$$

$$\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)=\left(-\frac{5}{3}\right)\times\left(-\frac{23}{9}-\frac{8}{3}\right)=\underline{\frac{235}{27}}$$

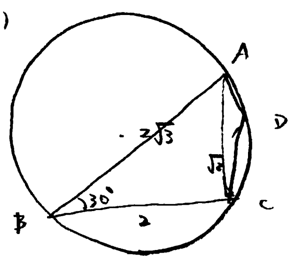
(2) $x^2+\frac{4}{3}x-\frac{4}{3}\leqq 0 \iff \left(x-\frac{2}{3}\right)(x+2)\leqq 0 \iff -2\leqq x\leqq \frac{2}{3}$

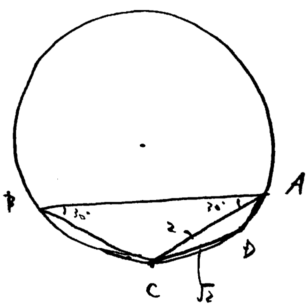
$|3x+1|\leqq 4 \iff -4\leqq 3x+1\leqq 4 \iff -\frac{5}{3}\leqq x\leqq 1$



$$\underline{-\frac{5}{3}\leqq x\leqq\frac{2}{3}}$$

(3)





$AC^2=(2\sqrt{3})^2+2^2-2\cdot2\cdot2\sqrt{3}\cos30^\circ=12+4-12=4$

$AC=2$

$\angle ADC=180^\circ-\angle ABC=150^\circ$

$\triangle ACD$について余弦定理より

$2^2=\sqrt{2}^2+AD^2-2\cdot\sqrt{2}\cdot AD\cdot\cos150^\circ$

$4=2+AD^2+\sqrt{6}AD$

$AD^2+\sqrt{6}AD-2=0$

$$AD=\frac{-\sqrt{6}\pm\sqrt{6+8}}{2}=\frac{-\sqrt{6}\pm\sqrt{14}}{2}$$

$AD>0$だから

$$\underline{AD=\frac{\sqrt{14}-\sqrt{6}}{2}}$$

(4) $x-2y-4=0$ と $2x+y-3=0$ の交点をもとめる

$x-2(3-2x)-4=0 \iff 5x=10 \iff x=2,\ y=-1$

これを3つ目の式に代入　$2m^2-3m-9=0 \iff (2m+3)(m-3)=0$

$$\underline{m=-\frac{3}{2},\ 3}$$

[2] (1) $2,2,3,3,3,5 \Rightarrow \dfrac{6!}{3!2!} = 60$

$2,2,3,3,5,5 \Rightarrow \dfrac{6!}{2!2!2!} = 90$

$2,3,3,3,5,5 \Rightarrow \dfrac{6!}{3!2!} = 60$　　　210通り

末尾が5のものは

$$\frac{5!}{3!2!} + \frac{5!}{2!2!} + \frac{5!}{3!} = 10 + 30 + 20 = 60\text{通り}$$

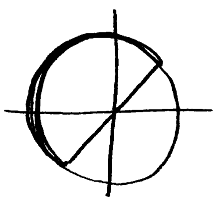
(2) $t^2 = 1 + 2\sin x\cos x \Leftrightarrow \sin x\cos x = \dfrac{t^2-1}{2}$

$$y = \sin^2 x\cos x + \sin x\cos^2 x + \sin x\cos x$$

$$= \sin x\cos x(\sin x + \cos x) + \sin x\cos x$$

$$= \frac{t^2-1}{2} \times t + \frac{t^2-1}{2} = \frac{1}{2}(t^2-1)(t+1) = \frac{1}{2}(t+1)^2(t-1)$$

$t = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$　　$\frac{\pi}{4} \leqq x + \frac{\pi}{4} \leqq \frac{5}{4}\pi$ より



$-\frac{1}{\sqrt{2}} \leqq \sin\left(x + \frac{\pi}{4}\right) \leqq 1$

$\underline{-1 \leqq t \leqq \sqrt{2}}$

$$y' = (t+1)(t-1) + \frac{1}{2}(t+1)^2 = \frac{1}{2}(t+1)(2t-2+t+1) = \frac{1}{2}(t+1)(3t-1)$$

t	-1	$\cdots$	$\frac{1}{3}$	$\cdots$	$\sqrt{2}$
y'	0	$-$	0	$+$	
y	0	↘	最小	↗	

$t = \frac{1}{3}$ のとき

$$y = \frac{1}{2} \times \left(\frac{4}{3}\right)^2 \times \left(-\frac{2}{3}\right) = -\frac{16}{27}$$

(3) $X^2 - 4\cdot\frac{1}{X} - 6 = 0 \Leftrightarrow \underline{X^3 - 6X - 4 = 0}$

$(X+2)(X^2-2X-2) = 0$

$X = -2,\ 1 \pm \sqrt{3}$

$X > 0$ だから　$X = 1 + \sqrt{3} = 2^x$　　$\therefore x = \underline{\log_2(1+\sqrt{3})}$

(4) $\alpha^2 - 5\alpha + 4 = 0 \iff (\alpha - 1)(\alpha - 4)$

$$S_{n+2} - S_{n+1} = 4(S_{n+1} - S_n)$$

$$\therefore \quad S_{n+1} - S_n = 4(S_n - S_{n-1})$$

$$= 4^2(S_{n-1} - S_{n-2})$$

$$= 4^3(S_{n-2} - S_{n-3}) = \cdots = 4^{n-1}(S_2 - S_1)$$

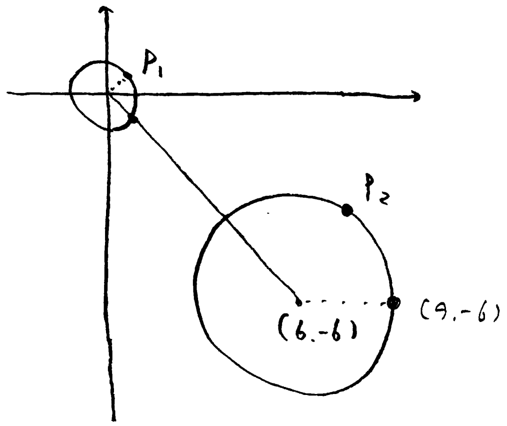
$$S_1 = a_1 = \frac{3}{4}, \quad S_2 = a_1 + a_2 = \frac{3}{4} + 3 = \frac{15}{4}$$

$$S_2 - S_1 = a_2 = 3$$

$$S_{n+1} - S_n = \underline{4^{n-1} \times 3}_{\,//}$$

$$a_6 = S_6 - S_5 = 3 \times 4^{5-1} = 3 \times 4^4 = \underline{768}_{\,//}$$

[3]



(1) 最短となるのは中心を結ぶ線分上にP_1, P_2があるとき $P_1\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$\overline{P_1P_2} = 6\sqrt{2} - 1 - 3 = \underline{6\sqrt{2} - 4}$

(2) P_1の偏角 $\frac{1}{2}\pi \times t = \frac{7}{4}\pi + 2n\pi$ （nは0以上の整数）

P_2 〃 $-\frac{7}{10}\pi \times t = -\frac{5}{4}\pi - 2m\pi$ （mは 〃 ）

が同時に成り立つとき最短となる

$-\frac{7}{5}\left(\frac{7}{4}\pi + 2n\pi\right) = -\frac{5}{4}\pi - 2m\pi$

$49\pi + 56n\pi = 25\pi + 40m\pi$

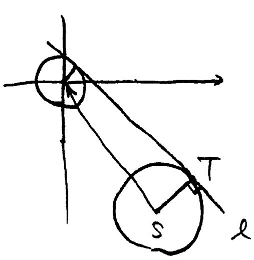
$56n\pi - 40m\pi = 24\pi$

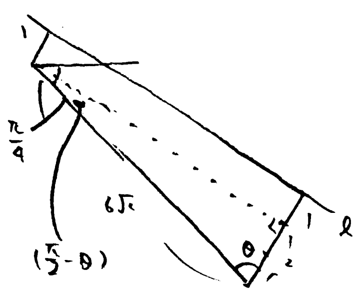
$7n - 5m = 3$

$n = 4,\ m = 5$ $\frac{1}{2}\pi \times t = \frac{7}{4}\pi + 8\pi$

$t = \frac{7}{2} + 16 = \underline{\frac{39}{2}}$

(3)





$\tan\theta = \frac{\sqrt{(6\sqrt{2})^2 - 2^2}}{2} = \frac{\sqrt{68}}{2} = \sqrt{17}$

$\tan\left(-\frac{\pi}{4} + \left(\frac{\pi}{2} - \theta\right)\right)$

$= \tan\left(\frac{\pi}{4} - \theta\right) = \frac{1 - \tan\theta}{1 + \tan\theta}$

$= \cdots$

[4]

(1) $\displaystyle\int_0^{\pi} \sin^3 x\,dx = \int_0^{\pi}(1-\cos^2 x)\sin x\,dx = \int_0^{\pi} \sin x + \cos^2 x(\cos x)'\,dx$

$\displaystyle = \left[-\cos x + \frac{1}{3}\cos^3 x\right]_0^{\pi} = +1 - \frac{1}{3} - \left(-1+\frac{1}{3}\right) = \underline{\frac{4}{3}}$

(2)

$\displaystyle C_1 = \int_{a_1}^{b_1} f_1(x)\,dx = \int_{\pi}^{(1+r)\pi} r\sin^3\left(\frac{1}{r}(x-\pi)\right)dx$

$\frac{1}{r}(x-\pi) = t$ とおくと $\quad \dfrac{dt}{dx} = \dfrac{1}{r}$,

x	$\pi \to (1+r)\pi$
t	$0 \to \pi$

$\displaystyle C_1 = \int_0^{\pi} r\sin^3 t \times r\,dt = r^2\int_0^{\pi}\sin^3 dt = \underline{\frac{4}{3}r^2}$

(3)

$\displaystyle C_n = \int_{a_n}^{b_n} f_n(x)\,dx$

$r^{-n}\left(x - \left(\dfrac{1-r^n}{1-r}\right)\pi\right) = t$ とおくと

$\dfrac{dt}{dx} = r^{-n}$,

x	$a_n \to b_n$
t	$0 \to \pi$

$\displaystyle C_n = \int_0^{\pi} r^n \sin^3 t \times r^n dt = r^{2n}\times\frac{4}{3} = \underline{\frac{4}{3}r^{2n}}$

(4)

$\displaystyle \lim_{n\to\infty}\sum_{k=1}^{n}\frac{4}{3}r^{2k} = \frac{\frac{4}{3}r^2}{1-r^2} = \underline{\frac{4r^2}{3(1-r^2)}}$

[5]

(1) $\dfrac{dx}{dt} = -2\sin t - 2\sin 2t = -2\sin t - 4\sin t\cos t = -2\sin t(1+2\cos t)$

$\dfrac{dy}{dt} = 2\cos t - 2\cos 2t = 2\cos t - 2(2\cos^2 t - 1) = 2(2\cos t+1)(1-\cos t)$

求める長さをLとすると

$$L = \int_0^{\frac{2}{3}\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

$$= \int_0^{\frac{2}{3}\pi} \sqrt{8 - 8(\sin t\sin 2t + \cos t\cos 2t)}\,dt$$

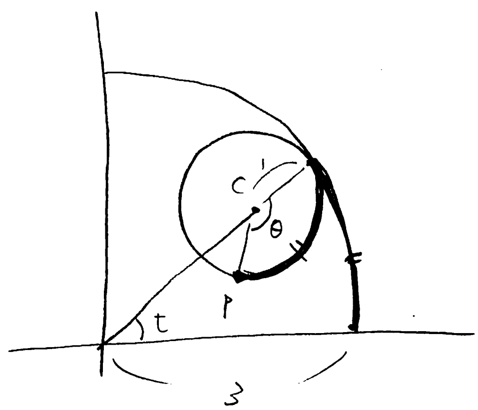
$$= 2\sqrt{2}\int_0^{\frac{2}{3}\pi} \sqrt{1-\cos t}\,dt$$

$$= 2\sqrt{2}\int_0^{\frac{2}{3}\pi} \sqrt{1 - 1 + 2\sin^2\frac{t}{2}}\,dt$$

$$= 2\sqrt{2}\int_0^{\frac{2}{3}\pi} \sqrt{2}\sin\frac{t}{2}\,dt$$

$$= 4\left[-\frac{2}{3}\cos\frac{t}{2}\right]_0^{\frac{2}{3}\pi}$$

$$= -8\left(\frac{1}{3} - 1\right) = \frac{16}{3}$$



(2) 右図の太線部の長さが等しいので

$3t = \theta$

Pの偏角は $+t - \theta = -2t$. よって (ア)は $\underline{2t}$

$\overrightarrow{CP} = (\cos(-2t), \sin(-2t)) = \underline{(\cos 2t, -\sin 2t)}$

$\overrightarrow{OC} = 2(\cos t, \sin t) = (2\cos t, 2\sin t)$

$\dfrac{dx}{dt} = 0$ となるのは $\sin t = 0$ または $\cos t = -\dfrac{1}{2}$ なので $t = 0, \pi, \dfrac{2}{3}\pi, \dfrac{4}{3}\pi, 2\pi$

$\dfrac{dy}{dt} = 0$ 〃 $\cos t = -\dfrac{1}{2}$ または $\cos t = +1$ 〃 $t = \dfrac{2}{3}\pi, \dfrac{4}{3}\pi, 0, 2\pi$

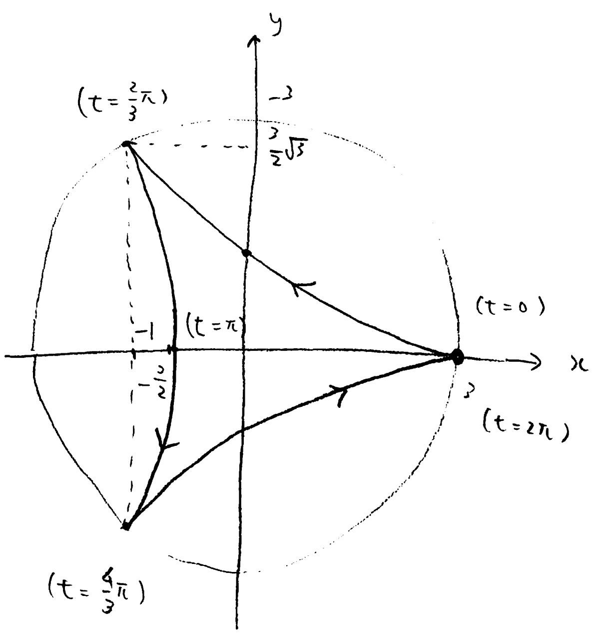
よって kの増減は次のようになる

t	0	$\cdots$	$\frac{2}{3}\pi$	$\cdots$	π	$\cdots$	$\frac{4}{3}\pi$	$\cdots$	2π
$\frac{dx}{dt}$	0	$-$	0	$+$	0	$-$	0	$+$	0
$\frac{dy}{dt}$	0	$+$	0	$-$	$-$	$-$	0	$+$	0
x	3	$\leftarrow$	$-\frac{3}{2}$	$\rightarrow$	-1	$\leftarrow$	$-\frac{3}{2}$	$\rightarrow$	3
y	0	$\uparrow$	$\frac{3}{2}\sqrt{3}$	$\downarrow$	0	$\downarrow$	$-\frac{3}{2}\sqrt{3}$	$\uparrow$	0
(x, y)		$\nwarrow$		$\searrow$		$\swarrow$		$\nearrow$	

-1

-1

$\sqrt{3}$ +



$$y^2 = 9 - \frac{9}{4} = \frac{27}{4}$$