①

(1) 同じ位置になるのは EまたはJ のとき.

$$\overset{E}{\frac{1}{6}\times\frac{1}{6}} + \overset{J}{\frac{1}{6}\times\frac{1}{6}} = \underline{\frac{1}{18}}_{\,//}$$

(2) A-J, J-I, J-E, D-E, E-F, E-J の6通り

$$\frac{1}{6}\times\frac{1}{6}\times 6 = \underline{\frac{1}{6}}_{\,//}$$

(3) 最大となるのは B-G, C-H

△BHG が直角三角形であることに注目して.

$$BH^2 + HG^2 = BG^2$$

$$(2\sqrt{3})^2 + 1^2 = BG^2$$

$$BG = \underline{\sqrt{13}}_{\,//}$$

確率は $\frac{1}{6}\times\frac{1}{6}\times 2 = \underline{\frac{1}{18}}_{\,//}$

(4). (i) P, Qが一致するとき.

(1)より $\frac{1}{18}$

(ii) PQO が一直線に並ぶとき.

$B-O_1-E$, $B-O_1-F$. $C-O_1-J$, $C-O_1-I$, $J-O_1-I$, $E-O_1-F$

$G-O_2-J$, $G-O_2-A$, $H-O_2-E$, $H-O_2-D$, $J-O_2-A$, $E-O_2-D$

$$\frac{1}{6}\times\frac{1}{6}\times\frac{1}{2}\times 12 = \frac{1}{6}$$

(i) (ii) より $\frac{1}{18} + \frac{1}{6} = \frac{4}{18} = \frac{2}{9}_{\,//}$

②

$b_1 = 1 \times a_1 = \underline{1}$

$a_n = \dfrac{b_n}{n}$, $a_{n+1} = \dfrac{b_{n+1}}{n+1}$ を漸化式に代入.

$$\frac{b_{n+1}}{n+1} = \frac{2n \times \frac{b_n}{n} + 1}{k(n+1)} \iff b_{n+1} = \underline{\frac{2}{k} b_n + \frac{1}{k}}$$

(1) $b_{n+1} - b_n = \dfrac{2}{k} b_n + \dfrac{1}{k} - b_n = \left(\dfrac{2}{k} - 1\right) b_n + \dfrac{1}{k}$

$\dfrac{2}{k} - 1 = 0$ のとき, 等差数列となるので $k = 2$.

このとき. $b_{n+1} - b_n = \dfrac{1}{2}$

$$b_n = b_1 + \frac{1}{2}(n-1) = \frac{1}{2}n + \frac{1}{2} = n a_n$$

$$\underline{a_n = \frac{1}{2} + \frac{1}{2n}}$$

(2) $\alpha = \dfrac{2}{k}\alpha + \dfrac{1}{k}$ を解いて. $\dfrac{k-2}{k}\alpha = \dfrac{1}{k}$ $\quad \alpha = \dfrac{1}{k-2}$

漸化式は

$$b_{n+1} - \frac{1}{k-2} = \frac{2}{k}\left(b_n - \frac{1}{k-2}\right)$$

$k-2$

$$b_n - \frac{1}{k-2} = \left(\frac{2}{k}\right)^{n-1} \times \left(b_1 - \frac{1}{k-2}\right)$$

$$b_n = \left(\frac{2}{k}\right)^{n-1}\left(\frac{k-3}{k-2}\right) + \frac{1}{k-2} = \underline{\frac{k-3}{k-2}\left(\frac{2}{k}\right)^{n-1} + \frac{1}{k-2}}$$

(3) $k-3=0$ のとき, すなわち $k = \underline{3}$

(4) $k=1$ のとき $b_n = 2^n - 1 = n a_n$ $\quad a_n = \dfrac{2^n - 1}{n}$

$$\lim_{n\to\infty} \frac{\frac{2^{n+1}-1}{n+1}}{\frac{2^n - 1}{n}} = \lim_{n\to\infty} \frac{\left(2 - \frac{1}{2^n}\right) \times 1}{\left(1 - \frac{1}{2^n}\right)\left(1 + \frac{1}{n}\right)} = 2$$

③

$a = 2\sqrt{5} \div 2 = \underline{\sqrt{5}}$ 　　 $a^2 - b^2 = 2^2$ より　$b = \underline{1}$

だ円の接線は　$\dfrac{x_1 x}{5} + y_1 y = 1.$　$\Leftrightarrow$　$y = -\dfrac{x_1}{5y_1}x + \dfrac{1}{y_1}$

$-\dfrac{x_1}{5y_1} = -\dfrac{1}{2}$ かつ　$\dfrac{x_1^2}{5} + y_1^2 = 1$ より.

$\dfrac{1}{5} \times \left(\dfrac{5}{2}y_1\right)^2 + y_1^2 = 1$　$\Leftrightarrow$　$y_1 = \pm\dfrac{2}{3}$, $x_1 = \pm\dfrac{5}{3}$

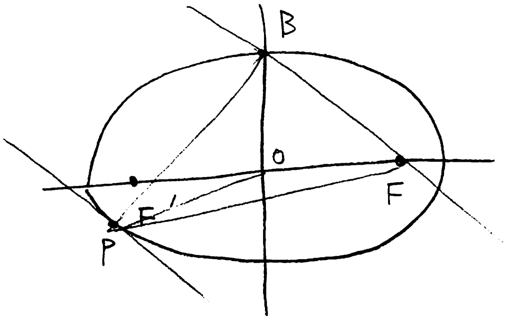
接線は
$\pm\dfrac{1}{3}x \pm \dfrac{2}{3}y = 1$　$\Leftrightarrow$　$\underline{x + 2y + 3 = 0 \text{ または } x + 2y - 3 = 0}$

BF の傾きは $-\dfrac{1}{2}$ なので.

右図の接線、すなわち接線が

$x + 2y + 3 = 0$ のとき最大.

接点は $\underline{\left(-\dfrac{5}{3}, -\dfrac{2}{3}\right)}$



このとき $\overrightarrow{PB} = \left(\dfrac{5}{3}, \dfrac{5}{3}\right)$

$\overrightarrow{PF} = \left(\dfrac{11}{3}, \dfrac{2}{3}\right)$

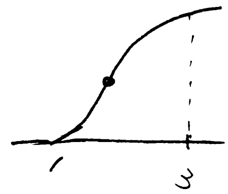
$\triangle PBF = \dfrac{1}{2}\left|\dfrac{5}{3} \times \dfrac{2}{3} - \dfrac{5}{3} \times \dfrac{11}{3}\right| = \underline{\dfrac{5}{2}}$

$\triangle OFP = \dfrac{1}{2} \times 2 \times \dfrac{2}{3} = \dfrac{2}{3}$

$\triangle OPB = \dfrac{1}{2} \times 1 \times \dfrac{5}{3} = \dfrac{5}{6}$ 　　　　 $\underline{4 : 5 : 6}$

$\triangle OBF = \dfrac{1}{2} \times 1 \times 2 = 1$

④



(1) $\displaystyle\int_1^3 x\sqrt{x^2-1}\,dx$

$= \frac{1}{2}\left[\ \frac{2}{3}(x^2-1)^{\frac{3}{2}}\right]_1^3 = \frac{1}{3}8^{\frac{3}{2}} = \frac{1}{3}\times 16\sqrt{2} = \underline{\frac{16\sqrt{2}}{3}}$ //.

(2) $y' = \sqrt{x^2-1} + x\times\cancel{\frac{1}{2}}\times\frac{\cancel{2}x}{\sqrt{x^2-1}} = \frac{2x^2-1}{\sqrt{x^2-1}}$

$y'' = \frac{4x\sqrt{x^2-1} - (2x^2-1)\cancel{\frac{1}{2}}\times\frac{\cancel{2}x}{\sqrt{x^2-1}}}{x^2-1} = \frac{4x^3-4x-2x^3+x}{(x^2-1)\sqrt{x^2-1}}$

$= \frac{2x^3-3x}{(x^2-1)\sqrt{x^2-1}}$

$2x^3-3x = x(2x^2-3) = 0$ となるのは $x = 0,\ \pm\frac{\sqrt{6}}{2}$

$x \geqq 1$ の範囲にあるのは $x = \frac{\sqrt{6}}{2}$.

このとき $y = \frac{\sqrt{6}}{2}\sqrt{\frac{3}{2}-1} = \frac{\sqrt{3}}{2}$ $\qquad (x, y) = \left(\frac{\sqrt{6}}{2}, \frac{\sqrt{3}}{2}\right)$.

(3) $x = x\sqrt{x^2-1}$ $\qquad x = \sqrt{2}$ $\qquad (x, y) = (\sqrt{2}, \sqrt{2})$

このとき 接線は

$y = \frac{2\sqrt{2}^2-1}{\sqrt{\sqrt{2}^2-1}}(x-\sqrt{2})+\sqrt{2} \iff \underline{y = 3x - 2\sqrt{2}}$ //

(4) $3x - 2\sqrt{2} = x\sqrt{x^2-1}$

$9x^2 - 12\sqrt{2}x + 8 = x^4 - x^2$

$x^4 - 10x^2 + 12\sqrt{2}x - 8 = 0$

$\underline{(x-\sqrt{2})^2(x^2+2\sqrt{2}-4) = 0}$ //

$$
\begin{array}{r|rrrrr}
 & & 1 & 2\sqrt{2} & -4 & \\
\hline
1\ \ -2\sqrt{2}\ \ 2 & 1 & 0 & -10 & 12\sqrt{2} & -8 \\
 & 1 & -2\sqrt{2} & 2 & & \\
\hline
 & & 2\sqrt{2} & -12 & 12\sqrt{2} & \\
 & & 2\sqrt{2} & -8 & 4\sqrt{2} & \\
\hline
 & & & -4 & 8\sqrt{2} & -8 \\
 & & & -4 & 8\sqrt{2} & -8 \\
\hline
 & & & & & 0
\end{array}
$$

$x = -\sqrt{2} \pm \sqrt{2+4} = \underline{\sqrt{6}-\sqrt{2}}$.

$(\because x \geqq 1)$.