① (1) $\log_2 a + \frac{2\log_2 a}{3} + \frac{5}{6\log_2 a} + \frac{\frac{1}{2}\log_2 a}{\log_2 a} + \frac{\log_2 a}{\frac{1}{2}\log_2 a} = 0$

$X + \frac{2}{3}X + \frac{5}{6X} + \frac{5}{2} = 0$

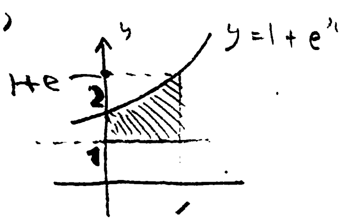
$10X^2 + 15X + 5 = 0$

$2X^2 + 3X + 1 = 0$

$(2X+1)(X+1) = 0$

$X = \log_2 a = -\frac{1}{2}, -1$ $\quad a = 2^{-\frac{1}{2}}, 2^{-1} = \frac{1}{\sqrt{2}}, \frac{1}{2}$ $\quad \therefore \underline{\frac{\sqrt{2}}{2}}$

(2)



$V = \int_0^1 \pi(1+e^x)^2 dx - \pi \times 1^2 \times 1$

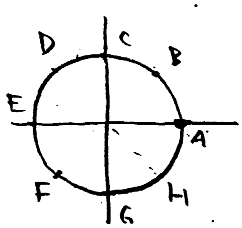
$= \pi\left[x + 2e^x + \frac{1}{2}e^{2x}\right]_0^1 - \pi$

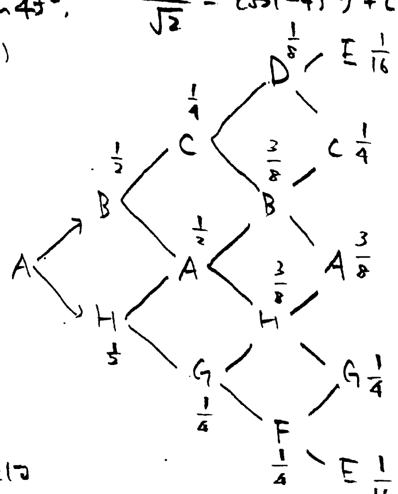
$= \pi\left(1 + 2e + \frac{1}{2}e^2\right) - \pi\left(0 + 2 + \frac{1}{2}\right) - \pi$

$= \underline{\pi\left(\frac{1}{2}e^2 + 2e - \frac{5}{2}\right)}$

(3) $36 \times \left(\frac{1}{6}\right)^2 = \underline{1}$

② (1) $\frac{1+i}{\sqrt{2}} = \cos 45° + i\sin 45°$ $(=\omega)$, $\quad \frac{1-i}{\sqrt{2}} = \cos(-45°) + i\sin(-45°)$ $(=\bar{\omega})$ とおく.





A→B→C→B→A または
A→H→G→H→A $\quad \left(\frac{1}{2}\right)^4 \times 2 = \underline{\frac{1}{8}}$

(2) $\frac{3}{8}$ $\left(= {}_4C_2 \times \left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2\right)$

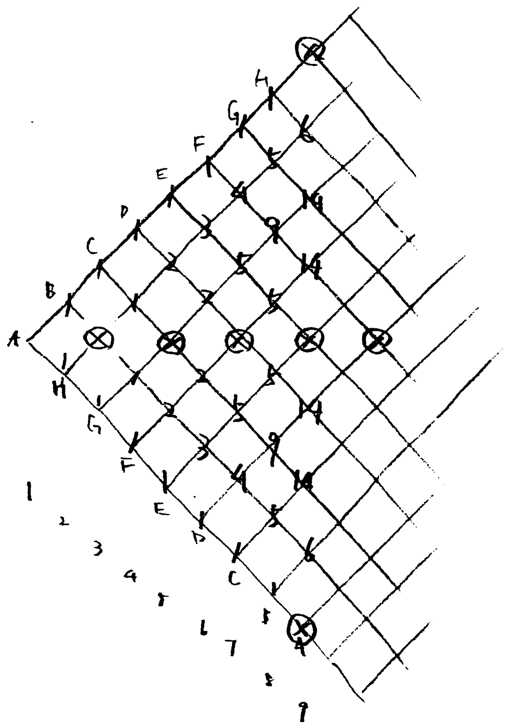
(3) $\omega\omega\omega\omega\bar{\omega}\bar{\omega}\bar{\omega}\bar{\omega}$

$\omega\omega\omega\omega\omega\omega\omega\omega$

$\bar{\omega}\bar{\omega}\bar{\omega}\bar{\omega}\bar{\omega}\bar{\omega}\bar{\omega}\bar{\omega}$

${}_8C_4 \cdot \left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^8 + \left(\frac{1}{2}\right)^8$

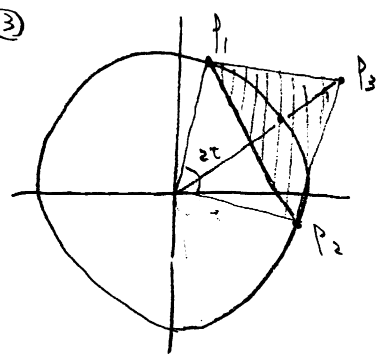
$= \frac{1}{256}\left(\frac{8 \cdot 7 \cdot 6 \cdot 5}{4 \cdot 3 \cdot 2} + 1 + 1\right) = \frac{72}{256} = \frac{18}{64} = \underline{\frac{9}{32}}$



$$\frac{(14+14+6)\times 2}{2^8} = \frac{17}{64}$$

$$1 - \frac{17}{64} = \frac{47}{64}$$

③



(1) $\overrightarrow{OP_1}\cdot\overrightarrow{OP_2} = 0$ だから $\angle P_1OP_2 = 90°$

$\therefore |\overrightarrow{P_1P_2}| = \sqrt{2}$

$$\begin{pmatrix} -\sin 2t - \cos 2t \\ \cos 2t - \sin 2t \end{pmatrix}\cdot\begin{pmatrix} \sqrt{2}\cos t - \cos 2t \\ \sqrt{2}\sin t - \sin 2t \end{pmatrix}$$

$$= -\sqrt{2}\cos t\sin 2t + \cancel{\cos 2t\sin 2t} - \sqrt{2}\cos t\cos 2t + \cos^2 2t$$
$$+ \sqrt{2}\cos 2t\sin t - \cancel{\cos 2t\sin 2t} - \sqrt{2}\sin t\sin 2t + \sin^2 2t$$

$$= \sqrt{2}(\sin t\cos 2t - \sin t\sin 2t - \cos t\sin 2t - \cos t\cos 2t) + 1$$

$$= \sqrt{2}(\sin(t-2t) - \cos(2t-t)) + 1$$

$$= -\sqrt{2}\sin t - \sqrt{2}\cos t + 1$$

$$= 1 - \sqrt{2}\sin t - \sqrt{2}\cos t$$

(2) $|\overrightarrow{P_1P_3}|^2 = (\sqrt{2}\cos t - \cos 2t)^2 + (\sqrt{2}\sin t - \sin 2t)^2$

$$= 2\cos^2 t + \cos^2 2t - 2\sqrt{2}\cos t\cos 2t + 2\sin^2 t + \sin^2 2t - 2\sqrt{2}\sin t\sin 2t$$

$$= 2 + 1 - 2\sqrt{2}\cos(2t-t) = 3 - 2\sqrt{2}\cos t$$

$$f(t) = |\overrightarrow{P_1P_2}|^2|\overrightarrow{P_1P_3}|^2 - (\overrightarrow{P_1P_2}\cdot\overrightarrow{P_1P_3})^2 = 2(3 - 2\sqrt{2}\cos t) - (1 - \sqrt{2}\sin t - \sqrt{2}\cos t)^2$$

$$= 6 - 4\sqrt{2}\cos t - 1 \underbrace{-2\sin^2 t - 2\cos^2 t}_{-2} + 2\sqrt{2}\sin t - 4\sin t\cos t + 2\sqrt{2}\cos t$$

$$= \underline{3 - 2\sqrt{2}\cos t + 2\sqrt{2}\sin t - 2\sin 2t}$$

(3) $f'(t) = 2\sqrt{2}\sin t + 2\sqrt{2}\cos t - 4\cos 2t$

$= 2\sqrt{2}(\cos t + \sin t) - 4(\cos^2 t - \sin^2 t)$

$= 2\sqrt{2}(\cos t + \sin t)(1 - \sqrt{2}\cos t + \sqrt{2}\sin t)$

$\cos t = -\sin t$ より $t = \frac{3}{4}\pi$

$2(\sin t - \cos t) = -1$

$\sin\left(t - \frac{\pi}{4}\right) = -\frac{1}{2}$ $\quad t - \frac{\pi}{4} = \frac{7}{6}\pi,\ -\frac{1}{6}\pi$ $\quad t = \frac{17}{12}\pi,\ \frac{1}{12}\pi$

t	$\frac{1}{12}\pi$	$\cdots$	$\frac{3}{4}\pi$	$\cdots$	$\frac{17}{12}\pi$
$f'(t)$	0	+	0	−	0
$f(t)$		↗		↘	

$t = \frac{3}{4}\pi$

(4) $f\left(\frac{3}{4}\pi\right) = 3 - 2\sqrt{2} \times \left(-\frac{1}{\sqrt{2}}\right) + 2\sqrt{2}\left(\frac{1}{\sqrt{2}}\right) - 2(-1) = 3 + 2 + 2 + 2 = 9$

$\triangle P_1P_2P_3 = \frac{1}{2}\sqrt{f\left(\frac{3}{4}\pi\right)} = \underline{\frac{3}{2}}$

④ $n = 4x + 7y$

(1)

$y \backslash x$	0	1	2	3	4
0	0	4	8	12	16
1	7	11	15	19	23
2	14	18	22	26	30
3	21	25	29	33	37

0	4	8	12	16	20	24	
1	5	9	13	17	21	25	
2	6	10	14	18	22	26	
3	7	11	15	19	23	27	

(2) y_0をaで割った商をy_1とする ($0 \leqq y_1 < y_0$)

$$y_0 = ay_1 + y$$

$$n = ax_0 + by_0 = ax_0 + b(ay_1 + y)$$

$$= a(x_0 + by_1) + by$$

よって $x = x_0 + by_1$ とすれば $n = ax + by$ と表せる.

(3) $(a-1)(b-1)-1$ がAの要素だと仮定すると.

$ax+by = n = (a-1)(b-1)-1$ を満たす x, y が存在する.

(2)より. yは $0 \leqq y < a$ と限定することができ、また同様に

xも $0 \leqq x < b$ とおくことができる.

$$ax + by = (a-1)(b-1) - 1$$

$$\Leftrightarrow \quad a(x-b+1) = -b(y+1)$$

ここで左辺はaの倍数だから右辺もaの倍数であるが.

a, bは互いに素なので $y+1$ がaの倍数でなければいけない

$0 \leqq y < a$ より. $y = a-1$ となり. このとき

$$a(x-b+1) = -ab$$

$$x - b + 1 = -b$$

$$x = -1$$

これは $0 \leqq x < b$ に矛盾する.

よって $n=(a-1)(b-1)-1$ としたとき nはAの要素ではない.

(4) $(a-1)(b-1) \geq 0$

n を a で割った余りを a'、商を a_1 とする

$a_1 \geq 0$, $0 \leq a' < a$

$n = aa_1 + a' = (a-1)(b-1)$