① (1) (i) $1+\tan^2\theta = \frac{1}{\cos^2\theta}$ より

$\cos^2\theta = \frac{1}{1+\tan^2\theta} = \frac{1}{1+(\frac{1}{6})^2} = \underline{\frac{36}{37}}$

$\sin 2\theta = 2\cos\theta\cdot\sin\theta = 2\cos^2\theta\cdot\tan\theta = 2\times\frac{36}{37}\times\frac{1}{6} = \underline{\frac{12}{37}}$

(2)(i) $\vec{OA}\cdot\vec{OB} = (\sqrt{2}, 2, 0)\cdot(\sqrt{2}, -1, \sqrt{3}) = 2-2-0=0$

$\vec{OB}\cdot\vec{OC} = (\sqrt{2}, -1, \sqrt{3})\cdot(\sqrt{2}, -1, -\sqrt{3}) = 2+1-3=0$

$\vec{OC}\cdot\vec{OA} = (\sqrt{2}, -1, -\sqrt{3})\cdot(\sqrt{2}, 2, 0) = 2-2-0=0$

$\therefore \vec{OA}\cdot\vec{OB} = \vec{OB}\cdot\vec{OC} = \vec{OC}\cdot\vec{OA} = \underline{0}$

(ii) $|\vec{OA}| = \sqrt{2+4} = \sqrt{6}$, $|\vec{OB}| = \sqrt{2+1+3} = \sqrt{6}$, $|\vec{OC}| = \sqrt{2+1+3} = \sqrt{6}$

題意の平行六面体は1辺の長さが$\sqrt{6}$の立方体なので体積は $V = \sqrt{6}^3 = \underline{6\sqrt{6}}$

(3) $x=y=z=0$のとき $m^2+m^2+m^2 = a^2m^2+b^2m^2+c^2m^2$

$\Leftrightarrow a^2+b^2+c^2=3$

$x=y=0,\ z=\sqrt{2}$のとき $\cancel{m^2}+\cancel{m^2}\cancel{-2m}+\cancel{1}+\cancel{m^2}\cancel{+2m}+\cancel{1} = \cancel{a^2m^2}+\cancel{b^2m^2}+\cancel{2}-2\sqrt{2}cm+\cancel{m^2c^2}$

$2\sqrt{2}cm = 0 \qquad c=0$

$x=z=0,\ y=\sqrt{6}$のとき $\cancel{m^2}-\cancel{4m}+4+\cancel{m^2}+\cancel{2m}+1+\cancel{m^2}+\cancel{2m}+1 = \cancel{a^2m^2}+\cancel{c^2m^2}+6-2\sqrt{6}bm+\cancel{b^2m^2}$

$\Leftrightarrow bm=0 \qquad b=0$

$y=z=0,\ x=\sqrt{3}$のとき $\cancel{1}-2m+\cancel{m^2}+\cancel{1}-2m+\cancel{m^2}+\cancel{1}-2m+\cancel{m^2} = \cancel{3}-2\sqrt{3}am+\cancel{(a^2+b^2+c^2)m^2}$

$\Leftrightarrow 2\sqrt{3}ma-6m=0. \qquad a=\frac{6}{2\sqrt{3}}=\sqrt{3}$

$a=\sqrt{3},\ b=c=0$のとき、

左辺＝右辺は常に成り立つので $a=\sqrt{3},\ b=c=0$.

② (1) $F(s) = -e^{-\frac{(s-3)^2}{2}}$

$F'(s) = -e^{-\frac{(s-3)^2}{2}} \times \left(-\frac{1}{2}\times 2(s-3)\right)$

$= (s-3)e^{-\frac{(s-3)^2}{2}}$

(2) $\int_{-x}^{x} e^{-\frac{(s-3)^2}{2}} ds = (*)$ とする

$\frac{s-3}{\sqrt{2}} = t$ とおく。

s	$-x \to x$
t	$\frac{-x-3}{\sqrt{2}} \to \frac{x-3}{\sqrt{2}}$

$\frac{dt}{ds} = \frac{1}{\sqrt{2}}$

となるので

$(*) = \int_{\frac{-x-3}{\sqrt{2}}}^{\frac{x-3}{\sqrt{2}}} e^{-t^2}\sqrt{2}\,dt = \sqrt{2}\int_0^{\frac{x-3}{\sqrt{2}}} e^{-t^2}dt - \underbrace{\sqrt{2}\int_0^{\frac{-x-3}{\sqrt{2}}} e^{-t^2}dt}$

〜〜部について、$-t = u$ とすると 〜〜部 $= \int_0^{\frac{x+3}{\sqrt{2}}} e^{-(-u)^2}(-du)$

となるので

$(*) = \sqrt{2}\,G\left(\frac{x-3}{\sqrt{2}}\right) + \sqrt{2}\,G\left(\frac{x+3}{\sqrt{2}}\right)$ //

$\lim_{x\to\infty}\int_{-x}^{x} e^{-\frac{(s-3)^2}{2}} ds = \lim_{x\to\infty}\left\{\sqrt{2}\,G\left(\frac{x-3}{\sqrt{2}}\right) + \sqrt{2}\,G\left(\frac{x+3}{\sqrt{2}}\right)\right\}$

$= \sqrt{2}\,\frac{\sqrt{\pi}}{2}\times 2 = \sqrt{2\pi}$ $\left(\because\ x\to\infty \text{ のとき } \frac{x\pm 3}{\sqrt{2}} \to \infty\right)$

(3) (1)より $F'(s) = se^{-\frac{(s-3)^2}{2}} - 3e^{-\frac{(s-3)^2}{2}}$

上式を$-x$からxまでの範囲で積分する

$\int_{-x}^{x} F'(s)ds = \int_{-x}^{x} se^{-\frac{(s-3)^2}{2}} ds - 3\int_{-x}^{x} e^{-\frac{(s-3)^2}{2}} ds$

$\left[F(s)\right]_{-x}^{x} = \int_{-x}^{x} se^{-\frac{(s-3)^2}{2}} ds - 3\sqrt{2}\left(G\left(\frac{x-3}{\sqrt{2}}\right) + G\left(\frac{x+3}{\sqrt{2}}\right)\right)$

$\int_{-x}^{x} se^{-\frac{(s-3)^2}{2}} ds = F(x) - F(-x) + 3\sqrt{2}\,G\left(\frac{x-3}{\sqrt{2}}\right) + 3\sqrt{2}\,G\left(\frac{x+3}{\sqrt{2}}\right)$ //

$\lim_{x\to\infty} F(x) = \lim_{x\to\infty}\left(-e^{-\frac{(x-3)^2}{2}}\right) = 0$, $\lim_{x\to\infty} F(-x) = \lim_{x\to\infty}\left(-e^{-\frac{(-x-3)^2}{2}}\right) = 0$ だから

$$\lim_{x\to\infty}\int_{-x}^{x} s e^{-\frac{(s-3)^2}{2}}\,ds = 0+0+3\sqrt{2}\times\frac{\sqrt{\pi}}{2}\times 2 = \underline{3\sqrt{2\pi}}$$

(4) $$\int_{-x}^{x}(s-3)F'(s)\,ds = \Big[(s-3)F(s)\Big]_{-x}^{x} - \int_{-x}^{x}F(s)\,ds$$

$$(x-3)F(x) = (x-3)\left(-e^{-\frac{(x-3)^2}{2}}\right)$$

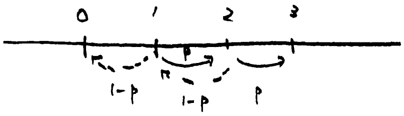
$$= -\sqrt{2}\times\frac{x-3}{\sqrt{2}}\times e^{-\left(\frac{x-3}{\sqrt{2}}\right)^2} \longrightarrow -\sqrt{2}\times 0 = 0$$

$$(-x-3)F(-x) = (-x-3)\left(-e^{-\left(\frac{x+3}{\sqrt{2}}\right)^2}\right)$$

$$= -\sqrt{2}\,\frac{x+3}{\sqrt{2}}\times e^{\left(\frac{x+3}{\sqrt{2}}\right)^2} \longrightarrow 0$$

よって $$\lim_{x\to\infty}\int_{-x}^{x}(x-3)F'(s)\,ds = 0+0+\sqrt{2\pi} = \underline{+\sqrt{2\pi}}$$

③



(1) $p_1(2) = p \times p = \underline{p^2}$

$p_1(4) = p_1(2) + p \times (1-p) \times p_1(2) = p^2 + p^3(1-p) = \underline{p^2(1+p-p^2)}$

(2) $p_1(2n+2) = p_1(2n) + (p(1-p))^n p^2$

$p_1(2n) = p_1(2) + \sum_{k=1}^{n-1} p^2(p-p^2)^k$

$= p^2 + p^2 \dfrac{1-(p-p^2)^{n-1}}{1-p+p^2} = \underline{\dfrac{p^2 - p^2(p-p^2)^n}{1-p+p^2}}$

(3) $p_2(1) = p$

$p_2(2n+1) = p_2(2n-1) + \{(1-p)p\}^n p$

$p_2(2n-1) = p + p\{(1-p)p\}^1 + p\{(1-p)p\}^2 + \cdots + p\{(1-p)p\}^{n-1}$

$= p \times \dfrac{p\{1-(p-p^2)^n\}}{1-p+p^2}$

(4) $\lim_{n\to\infty} p_1(2n) = \lim_{n\to\infty} \dfrac{p^2 - p^2\{-(p-\frac{1}{2})^2 + \frac{1}{4}\}^n}{1-p+p^2} = \dfrac{p^2}{1-p+p^2} = P_1$

($\because$ $0<p<1$ のとき $0 < -(p-\frac{1}{2})^2 + \frac{1}{4} < \frac{1}{4}$)

$\lim_{n\to\infty} p_2(2n) = \dfrac{p}{1-p+p^2} = P_2$

$k=1$ のとき $(1-p)P_0 + pP_2 - P_1 = (1-p)\times 0 + p \times \dfrac{p}{1-p+p^2} - \dfrac{p^2}{1-p+p^2} = \underline{0}$

$k=2$ のとき $(1-p)P_1 + pP_3 - P_2 = (1-p)\dfrac{p^2}{1-p+p^2} + p - \dfrac{p}{1-p+p^2}$

$= \dfrac{p^2 - p^3 + p - p^2 + p^3 - p}{1-p+p^2} = 0$